THE THERMAL CONDUCTIVITY OF SEDIMENTS
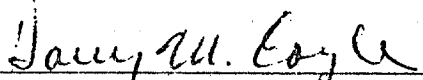
AS A FUNCTION OF POROSITY
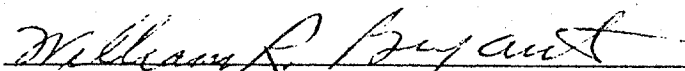
A Thesis

by

JAMES WARREN MILLER
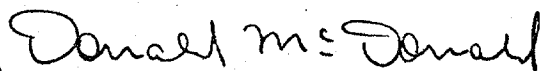
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August 1979

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JAMES WARREN MILLER

Submitted to the Graduate College of
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ABSTRACT

The Thermal Conductivity of Sediments

as a Function of Porosity

(August 1979)

JAMES WARREN MILLER, B.S., Bucknell University

Chairman of Advisory Committee: Dr. L. J. Thompson

A new test apparatus has been developed to determine the thermal
conductivity of sediments. A spherical transducer consisting of a
heater and a thermister is buried in the center of a soil sample at the
start of a consolidation test. The thermal conductivity of Kaolinite
clay was measured at various consolidation loads, and it was found that
the relationship between porosity and the thermal conductivity was
parabolic. The thermal conductivity of Kaolinite clay was considerably
lower than the thermal conductivity of other clays found in the
literature. It was decided that the thermal conductivity of clay
sediments is anistropic.

ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to Dr. Louis J. Thompson, Chairman of the author's committee, for his expert leadership and tireless guidance throughout this study.

Special thanks are also extended to Drs. Harry M. Coyle and William R. Bryant for their review and criticism in improving this thesis.

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INTRODUCTION

While drilling in the earth it is not unusual to encounter formations in which the pore water pressure exceeds the hydrostatic pressure. In some cases the pore water pressure may exceed even the overburden pressure. Formations that contain high pore water pressure and have a high permeability may cause a blowout of the well and even destroy the drilling rig as the water escapes. Due to the potential safety hazards posed by such formations and possible environmental pollution, detection is of major importance.

It has been theorized that the presence of overpressured zones may be detected by measuring fluctuations in the earth's temperature gradient. The temperature gradient in the earth is usually constant, but overpressured formations have been observed to have high temperature gradients (20). The average temperature gradient in the earth has been found to be around $1.1^{O}F/100$ ft ($2^{O}C/100$ m). Lewis and Rose studied the temperature gradients for the Texas Gulf Coast where overpressures exist, and found they ranged from $1.5^{O}F$ to $2.0^{O}F/100$ ft ($2.7^{O}C$ to $3.7^{O}C/100$ m). They theorized that the overpressured zones act as thermal barriers to heat and tend to insulate the surrounding material. Fig. 1 shows how the temperature gradient changes when a high pressured zone is encountered.

Assuming the flow of heat through any zone obeys Fourier's law, the following equation can be written:

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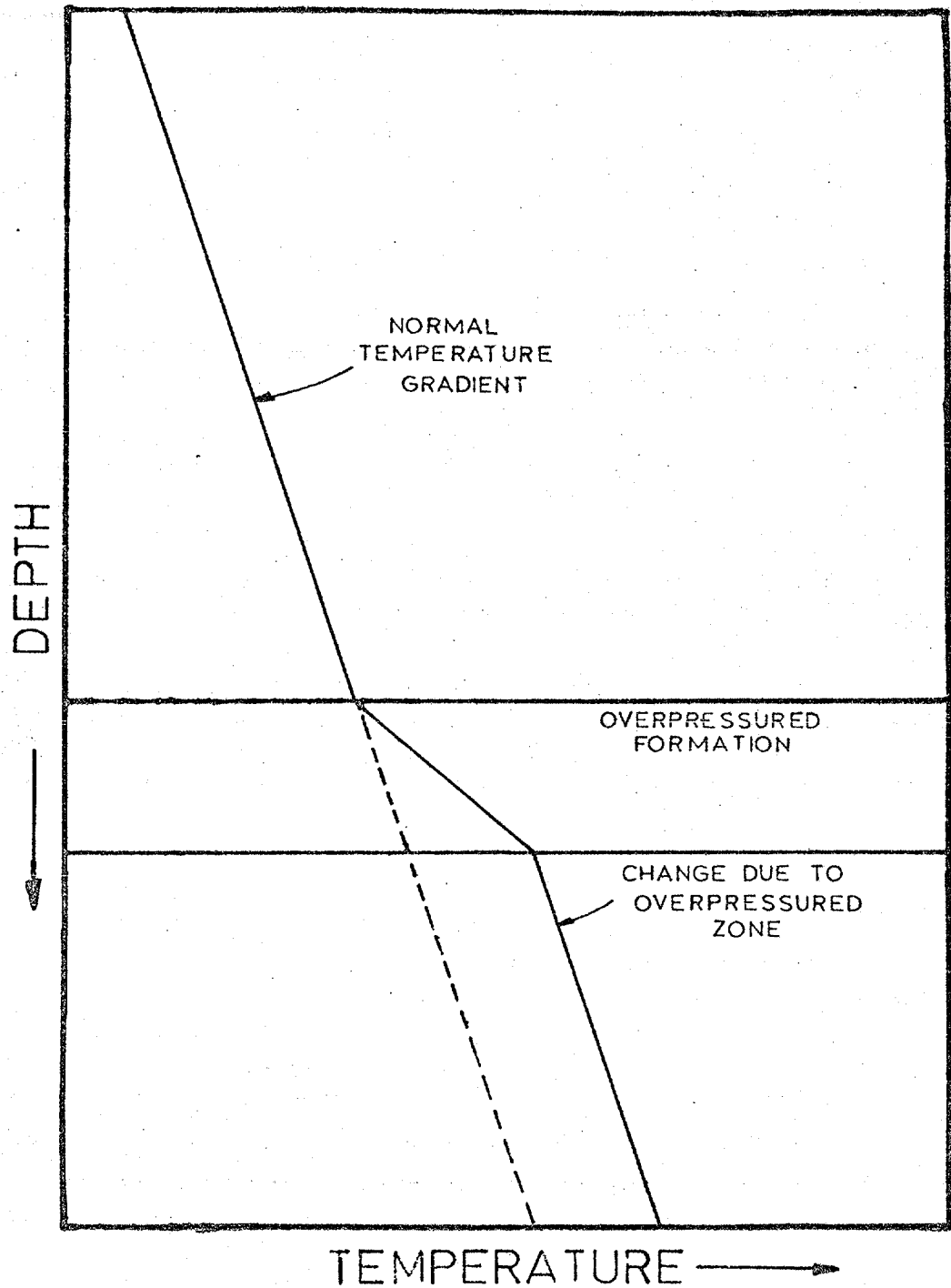
FIG.1.-Temperature gradient change
due to an overpressured formation

$$h = K \frac{\partial T}{\partial z} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \quad (1)$$

in which h = the heat flux or the amount of heat transmitted across a unit area per unit time; $\frac{\partial T}{\partial z}$ = the temperature gradient; K = thermal conductivity of the material, and z is the depth below the mudline. It can be seen from Equation 1 that if the heat flux, q, is constant and the temperature gradient, $\frac{\partial T}{\partial z}$, is increased as is the case for a high pressured zone, then the thermal conductivity, K, must decrease.

Lewis and Rose postulated that if a formation was to act as an insulator it would have a high porosity. This is primarily due to the individual thermal conductivity of each component of the soil-water matrix. Since the conductivity of water is approximately one-third the conductivity of the soil grains, the thermal conductivity of the matrix decreases as the porosity increases.

Colin Barker and others have theorized that the overpressured zones are isolated from surrounding sediments by impermeable barriers (3, 8, 21). Isolation of the pore water by an impermeable barrier can result in overpressures if the contained fluid is heated or an additional load is added to the fluid. Increases in pore water pressure can be caused by a displacement of a geologic formation downward. If the sediment is not isolated by an impermeable barrier, the hydrostatic pore water pressure would follow the relationship illustrated in Fig. 2 for a particular temperature gradient. Assuming hydrostatic conditions, Fig. 2 was developed by integrating with respect to depth an equation for the specific gravity of water when a linear thermal

STRESS IN KSI

CALCULATED DEPTH -1000 ft.

68° + 0.5°F/100ft
68° + 1.0°F/100ft
68° + 1.5°F/100ft
68° + 2.0°F/100ft
68° + 2.2°F/100ft

Thermal Gradient

a   b

Hydrostatic Pore Pressure

$$u = a + \frac{b}{10}$$

a = Hydrostatic Stress with constant Specific Gravity (1.0)

Stress Difference due to compressibility and Thermal
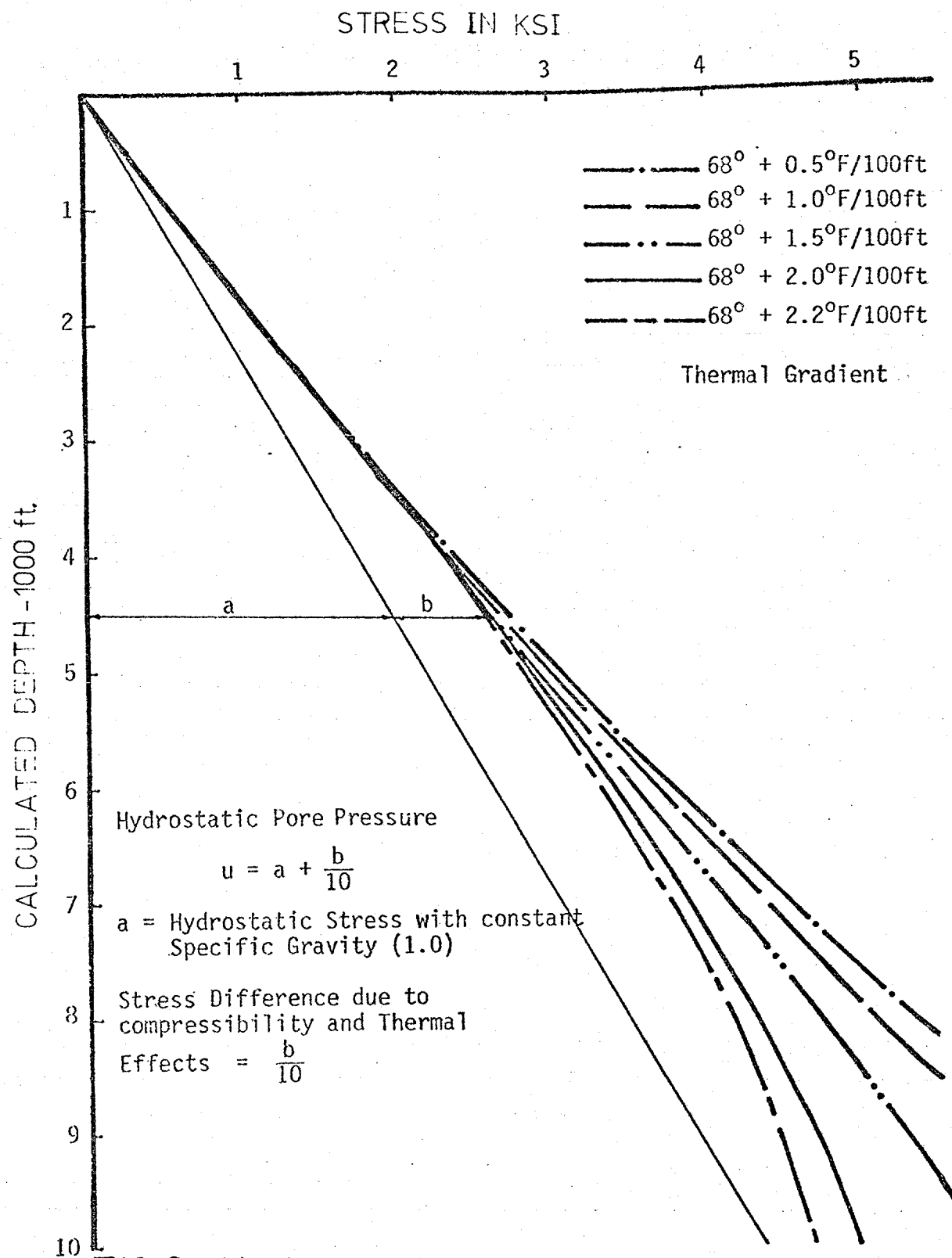
Effects = $\frac{b}{10}$

FIG.2.-Hydrostatic pressure for seawater with different thermal gradients(26)

gradient was assumed (26). The basis of the equation for the specific gravity of seawater (a function of pressure and temperature) is an equation of state for sea water developed by Fine, Wang and Millero of the University of Miami (12).

If the displaced sediment was surrounded by an impermeable barrier, the volume of the soil and water would remain constant. If the soil were to be displaced further into the earth, the increase in temperature would cause the pressure to increase as shown in Fig. 3. This figure relates the effect of pressure and temperature on the density of sea water. It was plotted from the equation of state by Fine, Wang and Millero (12). The following example will illustrate the use of these figures.

Assuming a temperature gradient of $1.4^oF/100$ ft ($2.5^oC/100$ ft), a unit volume of water 100 ft (305 m) below the mudline would have a temperature of 182^oF (28^oC) and a pressure of 450 psi (3100 kPa). This is shown in Fig. 2. The pressure is considered to be hydrostatic.

If this same unit volume is displaced to 9000 ft (2750 m), the hydrostatic pressure would increase to 4150 psi (28.6 MPa) and the temperature would increase to 194^oF (90^oC). However, had this same unit volume been displaced to 9000 ft (2750 m) without permitting any volume change and the temperature increased to the same 194^oF (90^oC), then as shown in Fig. 3, the pressure would become 5650 psi (38.9 MPa). Therefore under these conditions the excess pore pressure would be 1500 psi (10.3 MPa).

The temperature at any depth below the surface of the earth is a function of 1) the heat conducted through the crust due to cooling of
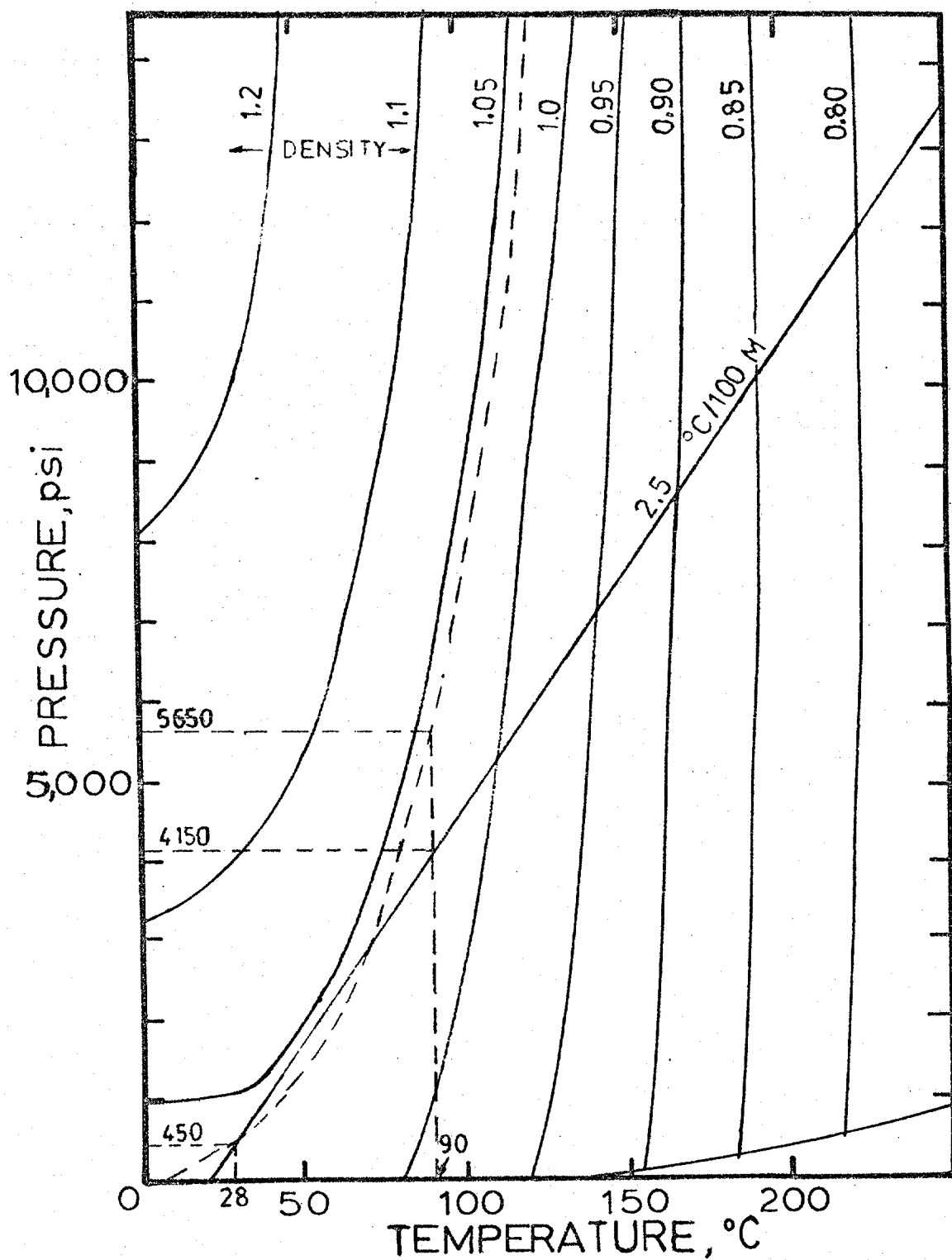
FIG. 3.- Pressure-temperature-density diagram for seawater based on equation of state (12)

the hot core, 2) the amount of heat that escapes from the surface to the atmosphere, 3) the thermal conductivity of the soil above and below the point where the temperature is measured, and 4) the amount of heat that is produced due to internal sources. Two methods of heat generation are present in the earth: compaction due to gravity and the decay of radioactive elements.

If the temperatures at different levels in the earth are known, the temperature gradient for that region can be found by dividing the temperature difference by the distance between the measured points. Bullard (6) determined the temperature gradient for various points in the Pacific Ocean by dropping a large probe, 15.5 ft (4.75 m) long and 1 in. (2.54 cm) in diameter, into the ocean floor. The probe contained two thermocouples to record temperatures, one at the probe tip and the other at the top of the probe. If the thermal conductivity of the material between the measured points is known, the amount of heat lost through the ocean bottom can be calculated using Fourier's law.

On land the temperature gradient is most often found by drilling a borehole and measuring the temperatures with increasing depth. This process can also be performed in the ocean. A typical example of temperature measurements at the bottom of many boreholes in one area is shown in Fig. 4. For this example a termperature gradient of approximately $1.8^{o}F/100$ ft ($3.3^{o}C/100$ m) was determined.

The amount of heat that flows out of the earth's surface has been estimated at 1.5 micro calories per cm^2 per sec (0.36 Btu per hr per ft^2). This is an average value computed from many different measurements made at various locations on the earth. The maximum amount of
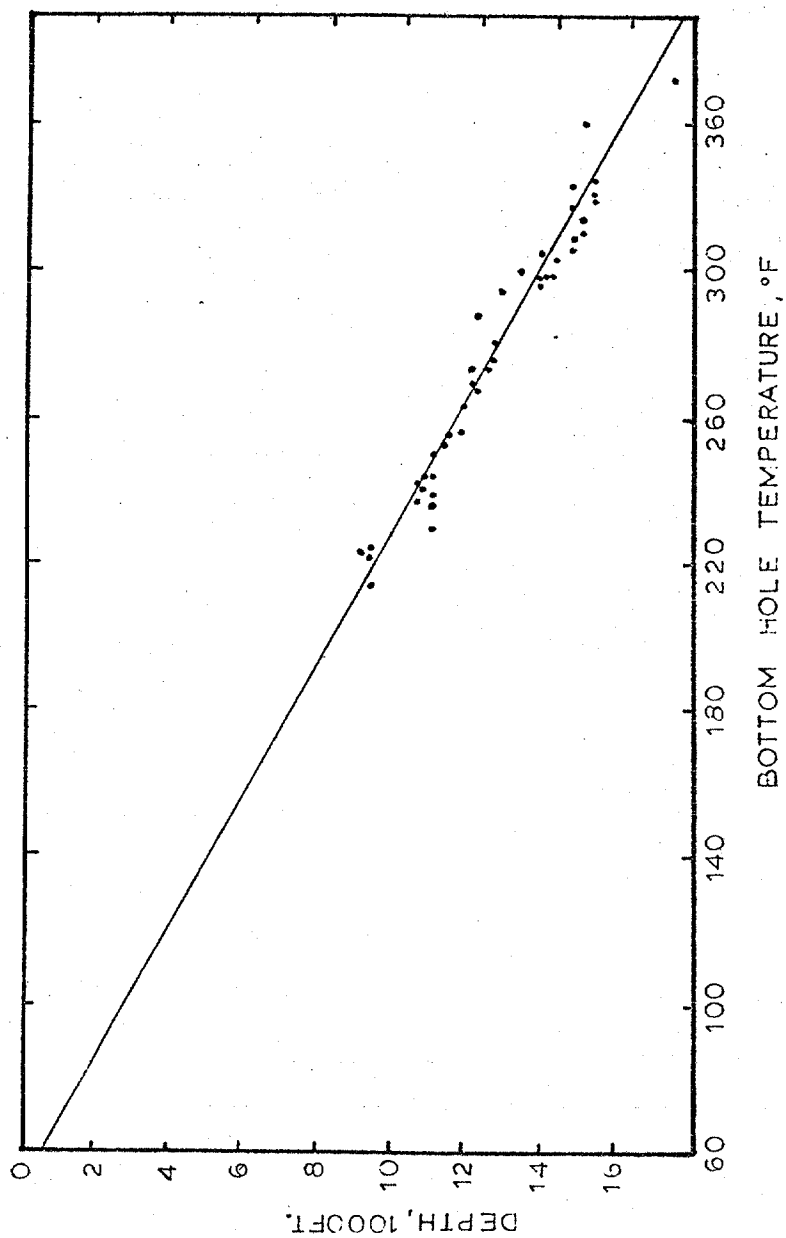
FIG.4.-Bottom hole temperatures in several wells in the Chocalate Bayou Field, Brazoria County, Texas(4)

heat escaping from the earth occurs at the mid ocean ridges in the
Atlantic and Pacific oceans. The heat flow from these ridges is 3.1
micro calories per cm^2 per sec (0.75 Btu per hr per ft^2) from the
East Pacific Rise and 2.9 (0.70) from the Mid-Atlantic Ridge. This is
twice the average value for the entire earth. A map of the heat flow
from the earth's surface is shown in Fig. 5.

It is generally assumed that heat flow in the earth is controlled
by conduction, although heat also could be transferred by convection.
Conduction transfers heat without any fluid flow, whereas convection
involves the displacement of fluid. Ribando and Torrance found that
the amount of heat transported by convection in a porous medium depends
upon the permeability. Due to the low permeabilities of clay sediments
it has been found that the transfer of heat due to convection is very
small (9, 18, 24).

Knowledge of geothermal gradients and high temperatures can also
be used to predict the location of oil and gas deposits. Oil and gas
formations are related to the temperature of the surrounding sediments.
Higher temperatures increase the rate at which oil will be produced.
A doubling of the surrounding temperature will double the rate at
which oil is formed. High geothermal gradients are usually found in oil
producing regions. This is due to the large quantity of heat that is
required to form the oil and gas deposits (2). High temperatures also
aid in the expulsion of oil and gas from a porous medium. Heat
increases the fluid pressure and causes a pressure potential which
causes fluid flow.

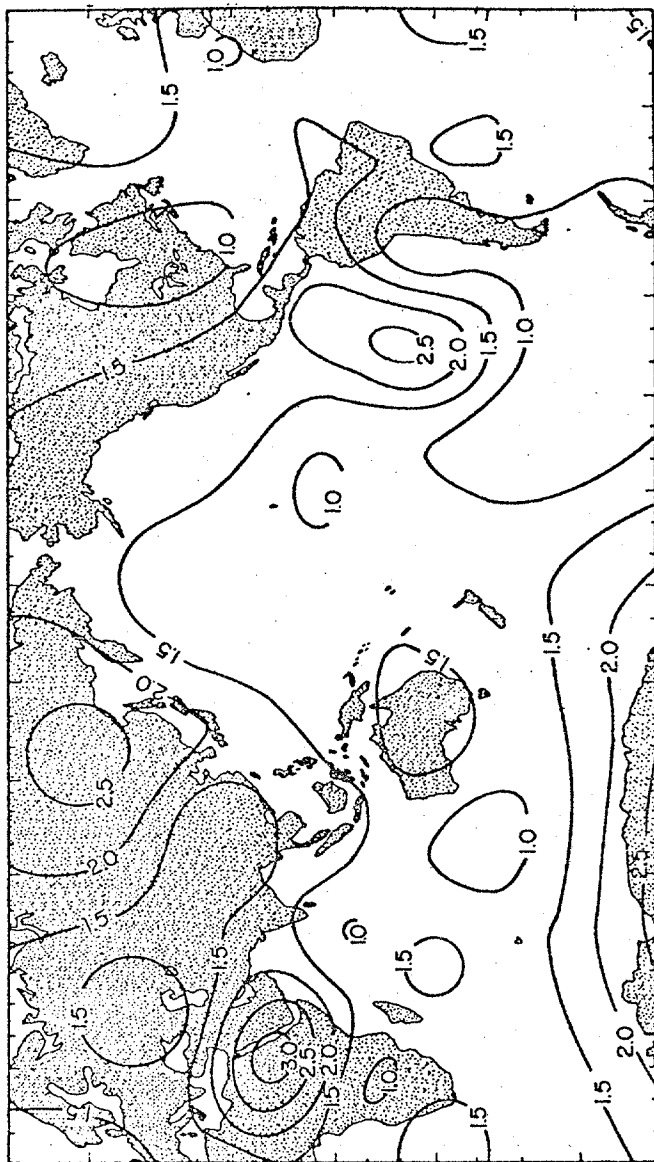Often overlooked in the development of overpressures is the

FIG.5-Contour map of heat flow, 10^{-6} cal/cm^2-sec (19)

mechanical process of sedimentation. If the permeability of the sediment decreases faster than the porosity, the pore water pressure will be locked into the sediment. As more sedimentation occurs, the pressure in the water will increase and will be stored as potential energy. Also as a sediment is compressed the irrecoverable strain energy is converted into heat. That heat which does not escape through either conduction or convection will raise the temperature of the sediment and the water contained. This is called the heat of compaction. This can also cause overpressures if the water is not allowed to escape.

Using the theory of thermodynamics of continuous media, the principles of conservation of energy, conservation of mass and the equations of motion can be applied to each component of the soil-water system (10). If this is done the following equations are obtained:

$$\left(\rho\dot{\xi} = \sigma\dot{\varepsilon} + \frac{\partial h}{\partial z} + \rho s\right)_{soil} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

and
$$\left(\rho\dot{\xi} = u\frac{\partial V}{\partial z} + \frac{\partial h}{\partial z}\right)_{water} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3)$$

where ρ = material density, $\dot{\xi}$ = rate of change in internal energy, σ = stress, $\dot{\varepsilon}$ = strain rate, u = pore pressure, $\frac{\partial V}{\partial z}$ = change in velocity of water with respect to depth, $\frac{\partial h}{\partial z}$ = change in heat flux vector with respect to depth and s = heat produced by internal sources. These are the reduced energy equations for soil and water (26). The rate of change of internal energy can be replaced by the rate of change of the thermal energy and the rate of change of recoverable strain energy (27):

$$\left(\rho \dot{\xi} = \sigma \dot{\varepsilon}_r + c_v \rho \frac{\partial \theta}{\partial t} \right)_{soil} \quad \cdots \cdots \cdots \cdots \cdots \quad (4)$$

and $$\left(\rho \dot{\xi} = u \frac{\partial V}{\partial z} + c_v \rho \frac{\partial \theta}{\partial t} \right)_{water} \quad \cdots \cdots \cdots \cdots \cdots \quad (5)$$

where $\dot{\varepsilon}_r$ = the rate of recoverable strain energy for the soil, c_v = the specific heat, $\frac{\partial \theta}{\partial t}$ = the change in temperature of the soil or water with respect to time and V = velocity of the water. The reduced energy equation can also be written for the combined soil-water system or Equations 2 and 3 can be added. If Equations 4 and 5 are substituted into Equations 2 and 3 and then added, an equation for the recoverable energy of the soil-water system is obtained. The term resulting from the addition of the heat flux terms, $\frac{\partial h_s}{\partial z} + \frac{\partial h_w}{\partial z}$, can be replaced by $\frac{\partial \left(k \frac{\partial T}{\partial z} \right)}{\partial z}$ assuming Fourier's law applies. The quantity $k \frac{\partial T}{\partial z}$ represents the heat flux through a combined soil-water system. The term resulting from the addition of the specific heat, $\left(c_v \rho_{soil} + c_v \rho_{water} \right) \frac{\partial \theta}{\partial t}$ can be replaced by $c_v \rho_{Total} \frac{\partial \theta}{\partial t}$. This quantity represents the change in internal energy due to heat only in a soil-water matrix. The equation obtained by adding Equation 2 and 3 is

$$c_{v_T} \rho_T \frac{\partial \theta}{\partial t} = \sigma \left(\dot{\varepsilon} - \dot{\varepsilon}_r \right) + \frac{\partial \left(k \frac{\partial \theta}{\partial z} \right)}{\partial z} + \left(\rho s \right)_{soil} \quad \cdots \cdots \cdots \quad (6)$$

Semantically Equation 6 can be written

Rate of Change of Thermal Energy	=	Irrecoverable Working	+	Divergance of the Heat Flow	+	Internal Power Supply

The specific heat of the sediment-water system can be determined from the porosity when the specific heat of the sediment particles and the specific heat of the water is known. The stress in the sediment-water system is a function of the porosity as determined from the experimental compressibility relationship (26). If the coefficient of thermal conductivity is determined experimentally as a function of porosity and the strains are written as functions of porosity, there will be two unknowns in Equation 6. These are the porosity and the temperature. Equation 6 will have to be combined with the other field equations before either porosity or temperature can be predicted as a function of depth and time.

Objective

The purpose of this research was to establish a relationship between the thermal conductivity and the porosity of various sediments.

THERMAL CONDUCTIVITY THEORY

The conduction of heat is a process in which heat flows from a
region of higher temperature to a region of lower temperature due to
the transfer of thermal energy at the particle to particle interface.
There is no mass movement of particles contained within the body to
another point in the medium. According to the kinetic theory, as a
particle gains energy it vibrates rapidly and rises in temperature. The
faster the particles vibrate is proportional to the temperature
rise in the medium. When particles, within a region of mass have
greater energy than surrounding particles, the particles of greater
energy will transmit part of their energy to the particles of lower
energy to maintain an energy balance. In the process of heat conduction
this is seen as a change in temperature. Energy flows in the form of
heat from regions of high temperature to regions of low temperature
until an equilibrium position is reached.

The fundamental law of heat conduction was postulated in 1822 by
James B. J. Fourier. He stated that the amount of heat flowing through
a slab of material by conduction can be expressed by the following
relationship:

$$dq = - kA \frac{d\theta}{dL} \quad \dots \dots \dots \dots \dots \dots \dots \dots \quad (7)$$

where dq = the amount of heat conducted per unit time, k = the thermal
conductivity of the material, A = the area perpendicular to the
direction of heat flow, and $\frac{d\theta}{dL}$ = the temperature difference measured
across an incremental length of the material. The thermal conductivity,

k, is a material property and has the units calorie per second per degree Celsius per centimeter ($\frac{cal}{sec-{}^oC-cm}$). The negative sign takes into account the fact that $\frac{d\theta}{dL}$ is negative, since the temperatures on the heated side of the slab are greater than on the opposite side and the energy flowing through the slab must be positive.

Consider a small element of material in a medium of the shape shown in Fig. 6. The element has dimensions dx, dy and dz. Neglecting external work and mechanical energy, an energy balance for the cube can be written in rectangular coordinates:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{s}{k} = \frac{1}{\alpha} \frac{\partial\theta}{\partial t} \quad\ldots\ldots\ldots\ldots\ldots \quad (8)$$

where $\frac{\partial^2 T}{\partial x^2}$ = the divergence of temperature with respect to the direction, x, s = rate of heat generated by internal sources, k = the thermal conductivity of the material, α = the thermal diffusivity and $\frac{\partial\theta}{\partial t}$ = the temperature rise of the material with respect to time. The thermal diffusivity, α, is equal to the thermal conductivity, k, divided by the specific heat, c_v, and the density of the material, ρ, or $\alpha = \frac{k}{\rho c_v}$. Equation 8 is the general heat conduction equation in three dimensions.

If there is no internal heat source, Equation 8 can be transformed to cylindrical or spherical coordinates, in which case the following equations are obtained.

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial \theta} \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$
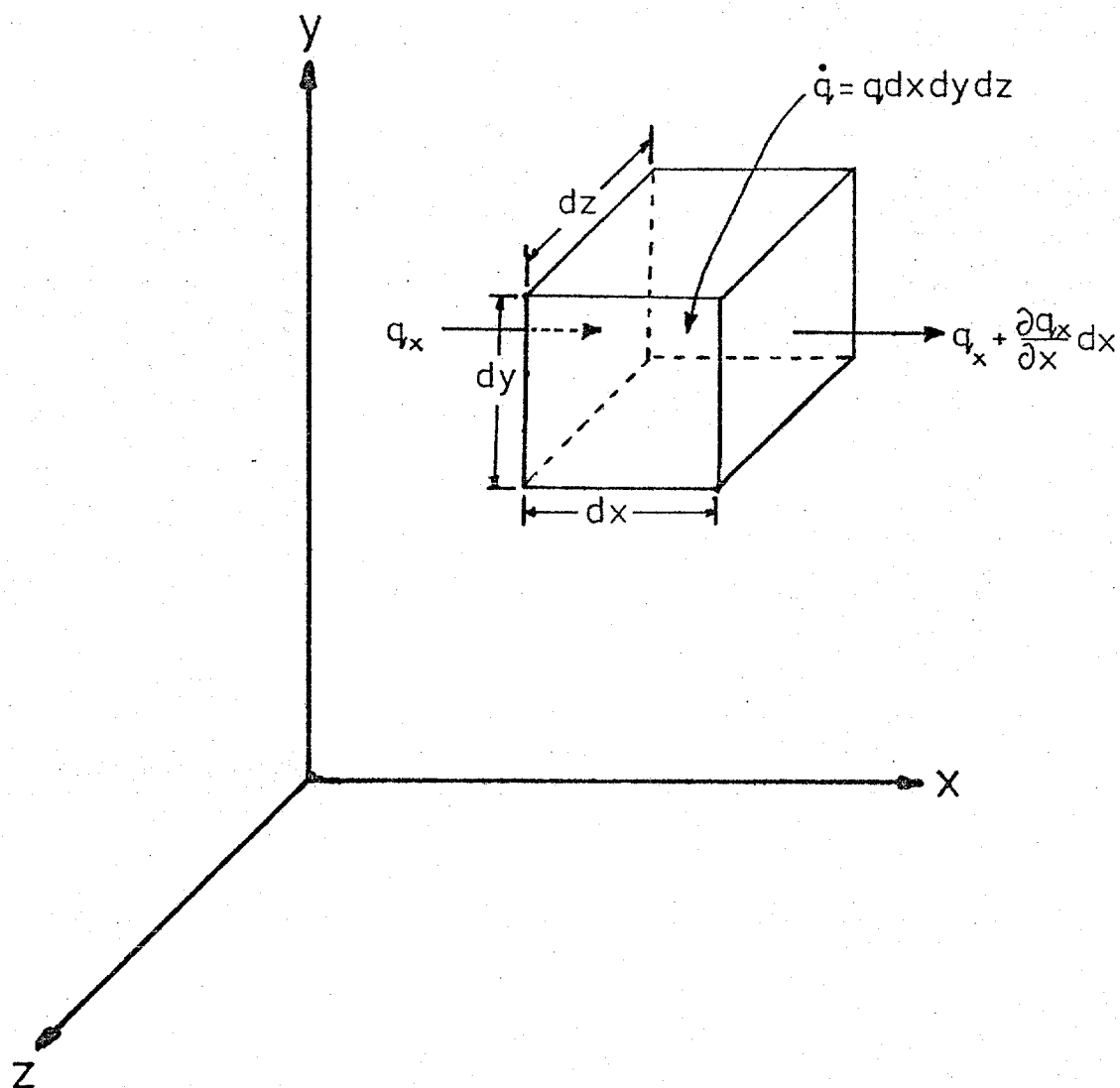
for spherical coordinates and

FIG.6.-Nomenclature for the derivation of the general heat-conduction equation in Cartesian coordinates

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial \theta} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (10)$$

for cylindrical coordinates. In these equations, r is the distance from the point liberating heat. These equations assume that the flow of heat is the same in all directions.

In the process of heat conduction there is a time during which the material absorbs energy until an equilibrium condition or final temperature is reached. The time during which the material absorbs heat is called the transient state. During this time the temperature at a point in the material is changing with time. The term in Equation 8, $\frac{1}{\alpha}\frac{\partial T}{\partial t}$, describes the transient part of the heat conduction equation. When the temperature change with respect to time is zero this term drops out and steady state heat flow results.

To determine the thermal properties of the soil, k and α, either a steady state or transient analysis can be used. Each method has certain advantages.

The determination of thermal conductivity by steady state techniques involves the solution of Equation 8, setting the right hand side to zero. Equation 8 then relates the temperature at specific points to the distance between these points. For a one dimensional problem Equation 7 can be used to solve for k. However, the steady state analysis suffers in three respects. First, heat losses must be carefully determined on all boundaries in order to know precisely the amount of heat, q, flowing through the area. If the heat losses are not measured with a high degree of accuracy, all error will be forced

into the value of the thermal conductivity. A second limitation arises when dealing with porous soils. Since water circulates easily in a porous medium, convection currents may be present. In this case, energy can also be transported by convection as well as by conduction. A third problem is the test duration. Determining the thermal conductivity by steady state methods can take several hours depending upon the value of thermal conductivity.

Using transient techniques, the problems encountered in the steady state technique are eliminated because the test duration is very short, usually only several minutes. However, when a transient method is employed, the solution of Equations 8, 9 or 10 generally involve complex techniques to obtain a particular solution. Carslaw and others (7, 13) give the solution for many problems involving numerous boundary conditions. If the transient technique is used, the solution generally has two unknowns, the specific heat, c_v, and the thermal conductivity, k, for the soil. The specific heat is usually obtained by the use of a calorimeter. However, for various minerals and water the specific heat is well known and the calorimeter is not needed. Using Equation 8, 9 or 10, if the specific heat is known, the thermal conductivity can be determined if the change in temperature with respect to time is measured at some given point.

For this study the spherical heat equation was used. Jaeger has solved Equation 9 for a sphere of radius, r, surrounded by a material of thermal conductivity, k. The equation describes the temperature rise of the sphere with respect to time if a given amount of energy is supplied to the sphere (7). The equation is:

$$\Delta T = \frac{q}{4 \cdot \Pi \cdot k \cdot r} \left\{ 1 - e^{\left(\frac{\alpha t}{r^2}\right)} \cdot \text{erfc} \sqrt{\left(\frac{\alpha t}{r^2}\right)} \right\} \quad \ldots \ldots \ldots \ldots \quad (11)$$

where ΔT = temperature rise of the sphere after time, t, q = rate of energy input to the sphere, k = thermal conductivity of the material surrounding the sphere, r = radius of the sphere and α = the thermal diffusivity of the surrounding material. The term "erfc" is the complimentary error function and is the solution to the integral

$$\text{erfc} \sqrt{\frac{\alpha t}{r^2}} = \frac{1}{2\Pi} \int_{\sqrt{\frac{\alpha t}{r^2}}}^{\infty} e^{-\beta^2} \, d\beta \quad \ldots \ldots \ldots \ldots \ldots \quad (12)$$

where $\beta = \frac{r}{2} \sqrt{\alpha t}$. The solution to this integral can be found in various mathematics and statistics books (1). If the temperature rise of the sphere and the specific heat and density of the material are known, then the thermal conductivity of the surrounding material can be found from Equation 11.

Equation 11 is the transient solution to the spherical heat flow equation. If large values for time are used the exponential and error function terms drop out and the equation becomes:

$$\Delta T = \frac{q}{4 \cdot \Pi \cdot k \cdot r} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (13)$$

This is the steady state equation for heat flow from a spherical source of radius, r, embedded in a material of infinite extent and thermal

conductivity, k.

PRESENT STATUS

Measurements of heat flowing through the floor of the ocean require an accurate value of the thermal conductivity of the sediment where the thermal gradient is measured. However, conventional measurements of thermal conductivity only give an indication of this quantity, since the sample will normally be disturbed by the measuring and sampling devices. The overburden pressure and pore pressures are usually changed when the measurements are taken.

Several devices have been constructed to determine the thermal conductivity of soil samples. All of these methods utilize a heating element and several thermisters to measure the temperature change in the soil. If the amount of heat flowing into the soil is known and the boundary conditions and geometric properties have been taken into account, the thermal conductivity of the soil can be calculated.

Ratcliffe conducted thermal conductivity tests on samples from the Pacific, Atlantic and Mediterranean areas using a hot plate device (23). Basically the mechanism consisted of a disc sandwiched between two layers of soil with thermocouples connected on each end of the soil, as seen in Fig. 7. The water cooled cold plates were used to measure the heat flowing out of the sample. The values for thermal conductivity were calculated from steady state equations. Water content of each sample was determined and plotted versus the thermal conductivity. It was found that the thermal conductivity decreased as the water content increased. It was also concluded that the effect of pressure on the
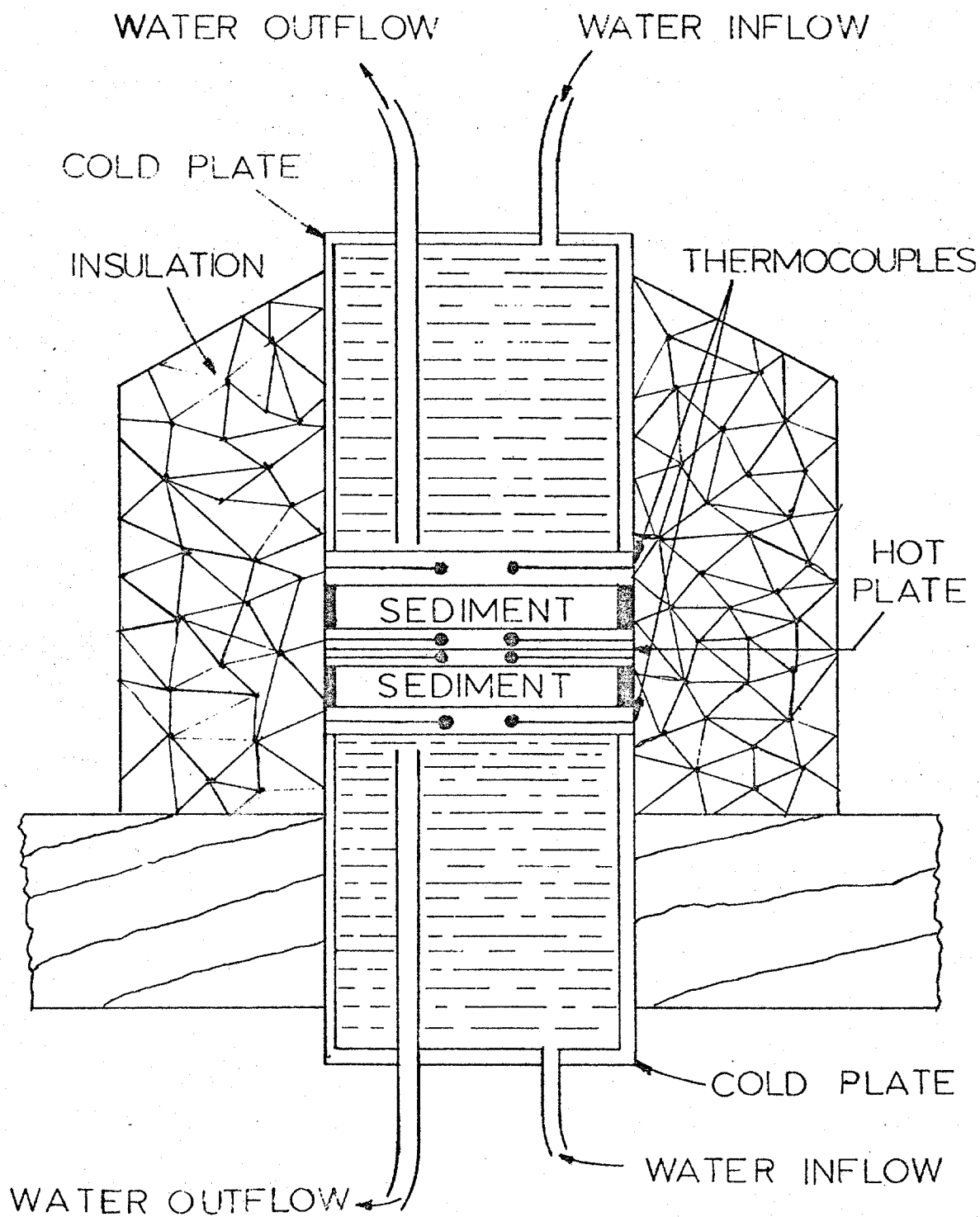
FIG. 7.-Hot plate apparatus used to determine the thermal conductivity of soils(23)

thermal conductivity was small, if not negligible. It was thought actual compression of the soil grains would not influence thermal conductivity.

Various people have also used other steady-state techniques to determine the thermal conductivity of soils. Using either cylindrical or spherical coordinates, an apparatus can be constructed to conform to either coordinate system. In either coordinate system, the temperature difference between two points is measured and the thermal conductivity is calculated from the appropriate equation. Generally, 10-12 hours is required to achieve steady state conditions.

The most common method in use today to determine the thermal conductivity of soil samples utilizes the transient part of the heat conduction equation. In this method, a needle probe of finite length and diameter is inserted into the soil sample. A linear heating element contained in the needle is turned on and the soil sample is heated. After 10 or 15 minutes the heater is turned off and the cooling of the needle is recorded by a temperature sensing device also contained inside the needle. A plot of temperature vs. time is then constructed. The needle probe approximates an infinite line source in an infinite medium. The solution to the line source is obtained from the cylindrical heat conduction equation for which Jaeger has obtained the solution. The temperature rise of the probe is given by the following equation:

$$\Delta T = \frac{q}{4 \cdot \pi \cdot k} \log \frac{t_1}{t_2} \quad \cdots \cdots \cdots \cdots \cdots \cdots \quad (14)$$

where ΔT = temperature rise of the probe between time t_1 and t_2, q = energy input to the probe and k = the thermal conductivity of the material surrounding the probe. If the temperature rise of the probe versus time is plotted on semi-log paper, the points will form a straight line. The thermal conductivity can then be calculated from the slope of this line, since the slope is equal to $\frac{q}{4 \cdot \pi \cdot k}$. It can be seen from Equation 14 that the thermal conductivity is independent of the probe radius and the specific heat of the material. This is an advantage for the needle probe.

There are many needle probes that have been constructed to determine the thermal conductivity of soils. Winterkorn built one that is 4 ft (1.2 m) long and 3/16 to 3/8 in. (0.5 to 1 cm) in diameter (31). Contained in the needle probe are the heating element and three thermocouples spaced evenly throughout the length of the probe. A borehole is drilled to allow the probe to be placed in the ground without damaging the needle. After the needle has been placed in the ground, the horizontal thermal conductivity is determined.

Another probe similar to Winterkorn's has also been used extensively to measure the thermal conductivity of soils. Stalhane and Pyke initially developed the probe and various people have refined it over the years (5, 17, 29). The probe is a number 20 gauge hypodermic needle 2.5 in. (6.4 cm) in length. Confined in the needle are a heating element and one thermister, as shown in Fig. 8. Using this method thermal conductivity of soil samples can be determined in times as short as ten minutes.

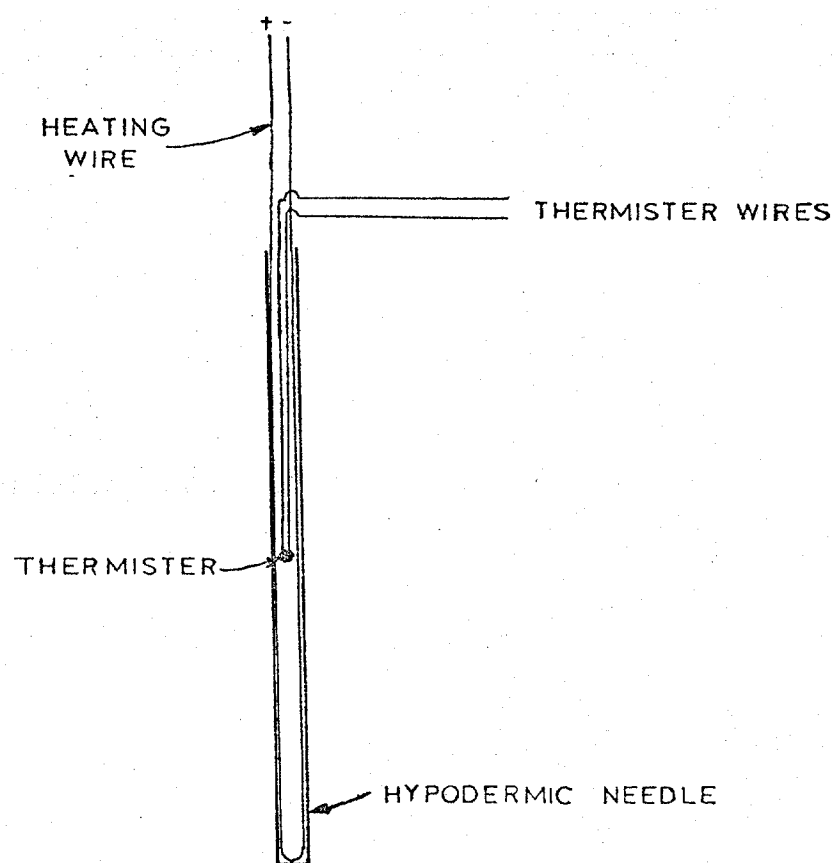The accuracy of the needle probe technique is somewhat limited by

FIG.8.-Needle Probe(28)

the following: its finite dimensions tend to distort the temperature readings, and the finite radius of the probe make early readings of temperature non-linear, such that the early readings are not used. The finite length of the probe also causes a heat loss at the end of the needle. This causes a temperature gradient to be established along the side of the probe. However, this effect is relatively small and careful use of the probe will limit this from happening. It measures the heat conductivity only in the planes perpendicular to the axis of the needle.

Shannon and Wells developed another technique to determine the thermal conductivity of soils (25). It involves measuring the temperature change at the center of a soil sample initially at $104^{\circ}F$ $(40^{\circ}C)$, after the sample is immersed in a $68^{\circ}F$ $(20^{\circ}C)$ water bath. The temperature change with respect to time is recorded and the thermal conductivity is calculated from the thermal diffusivity. The thermal diffusivity is obtained from a solution of the cylindrical heat conduction equation relating the temperature change at the center of a cylindrical sample to a dimensionless time factor. Shannon and Wells assumed the specific heat for the soils tested to be 0.2 cal/gr-$^{\circ}$C when in reality the specific heat is a function of porosity.

Ratcliffe and others have concluded that the thermal conductivity of saturated soils depends primarily upon moisture content or porosity. The majority of the heat is transported by the water, and the minerology of the soil does not influence the thermal conductivity very much. Fig 9 illustrates how the thermal conductivity varies with porosity. The data is from Ratcliffe (23), Kersten (15) and Von Herzon (29).
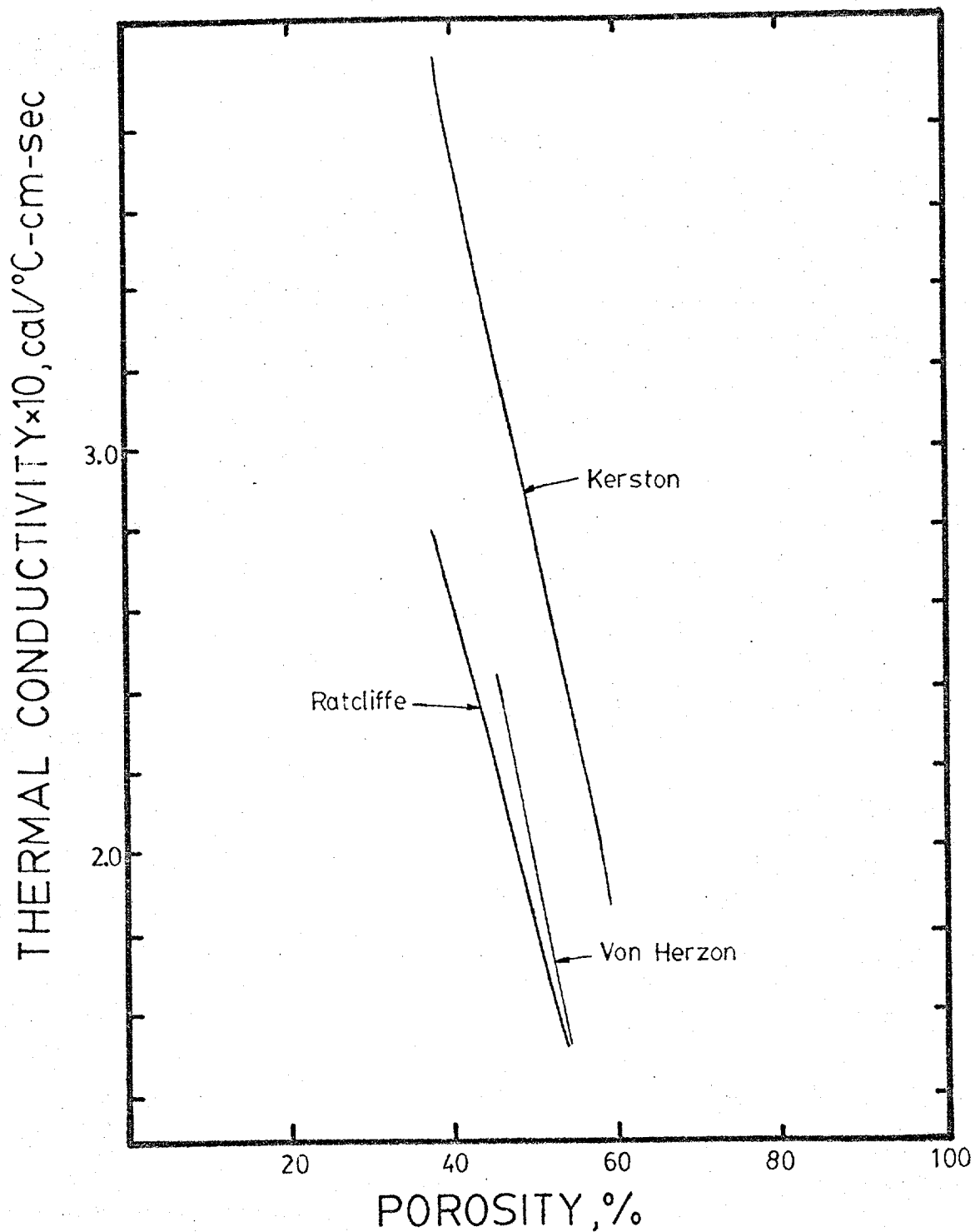
FIG.9.-Thermal conductivity versus porosity from other people as noted

EXPERIMENTAL TECHNIQUE

Temperature changes and the amount of energy input to the soil must be known if the thermal conductivity of soils is to be determined. These values must be known accurately if reliable values for the thermal conductivity of the material are to be obtained. As mentioned previously, there are two methods of analysis that can be used to determine thermal conductivity: steady state and transient techniques. Due to the amount of time required to complete a single test and because of the possibility for moisture migration, the steady state method of analysis was not used for this experiment. For this study, the transient method was chosen to determine the thermal conductivity. The transient method did not involve measuring the temperature distribution on the sample boundaries which is required in the steady state analysis. Also, when using steady state methods, elaborate instrumentation is needed or careful insulation must be applied to keep heat losses to a minimum. This is eliminated when a transient method is used.

The thermal conductivity of the soil-water matrix was measured at the time when the consolidation under a given load increment was complete. A transducer that could be easily adapted to the consolidation apparatus was devised. A device similar to the needle probe would have been sufficient if it could have been installed in the consolidometer. At pressures as high as 10,000 psi, as used in these tests, it would have been very difficult to insert the probe into the side of the sample after each consolidation test. This would have also disturbed the sample and the consolidation process.

It was finally decided to bury a small spherical transducer in

the soil sample at the start of the sequence of consolidation tests. The transducer was originally placed in the middle of the soil sample, and it was found that at the end of the test the transducer was still in the middle of the sample even though the height changed from 2 in. to 1/2 in. (5.1 to .14 cm).

The transducer used was constructed out of flat, brass foil, 7/16 in. (1.1 cm) in diameter. Two SR-4 strain gages orientated 90° apart were glued on one side of the probe. The strain gages acted as a heater when a constant voltage was applied. To measure the temperature change of the transducer, a small bead thermister was glued to the opposite side. As the temperature of the transducer increased, the change in resistance of the thermister was recorded by a strip chart recorder. The resistance of the thermister at any instant of time could then be converted to temperature by using a calibration curve of thermister resistance versus temperature. Once the resistances were converted to temperatures, a series of points for the temperature rise of the transducer versus time was obtained. This is very similar to the technique used for the needle probe.

The transducer used in this study approximated a sphere of known radius, r, embedded into an infinite medium. As stated earlier Jaeger has solved this spherical heat conduction problem so that the temperature rise of the transducer is given by

$$\Delta T = \frac{q}{4 \cdot \pi \cdot k \cdot r} \left\{ 1 - e^{\left(\frac{\alpha t}{r^2}\right)} \cdot \text{erfc} \sqrt{\left(\frac{\alpha t}{r^2}\right)} \right\} \quad \dots \dots \dots \dots \quad (11)$$

In actuality, the transducer used is not a perfect sphere but a disk. To use Equation 11 an effective radius for the disc had to be found.

To determine the effective radius r_{eff}, the transducer was buried into a material of known thermal properties. Paraffin wax with $k = 0.579$ x $10^{-3} \frac{cal}{sec \cdot {}^oC}$ and $c_v = .69 \frac{cal}{gr \; {}^oC}$ was chosen. Using Equation 11 the effective radius of the disc was calculated and found to be 0.17 in. (0.43 cm). To be sure this value was correct a second material of known properties was also studied. The second material, Ottawa sand with $k = 6.5$ x $10^{-4} \frac{cal}{sec \cdot {}^oC \cdot cm}$ and $c_v = 0.2 \frac{cal}{gr \; {}^oC}$, (16, 28) gave an effective radius of .15 in. (0.39 cm). Averaging the two values, an effective radius of 0.16 in. (0.41 cm) was obtained.

From Equation 11, it is found that there are actually three unknowns, k, c_v and ρ since $\alpha = \frac{k}{\rho c_v}$. It is not possible to solve for the thermal conductivity of the soil knowing only the transducer temperature versus time.

Knowing the porosity, the product of the total specific heat, c_{v_T}, and total density, ρ_{v_T}, for a soil can be found if the specific heat and specific gravity of the individual component phases, water and solids, is known (29, 30). Consider the soil-water system in Fig. 10. If the total volume of the system is one cubic centimeter, then the volume of water is the porosity ratio, n, and the volume of solids is 1-n. If the individual volumes are multiplied by their unit weights and specific heats respectively, added and then divided by the total volume, an expression for the total density times the total specific heat is obtained. Mathematically, this becomes

$$\rho_T c_{v_T} = \frac{n \gamma_0 G_{sw} C_{sw} + (1-n) G_s \gamma_0 C_s}{V_t} \quad \ldots \ldots \ldots \ldots \quad (15)$$

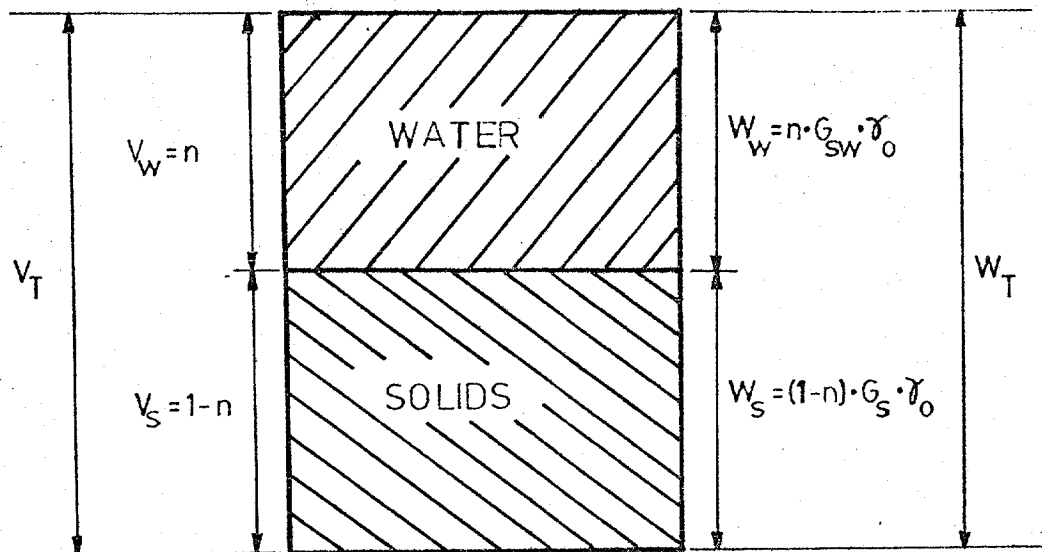where ρ_T = total density, c_{v_T} = total specific heat, n = porosity in

FIG.10.-Phase diagram for soil-water system

decimal form, γ_0 = unit weight of water, G_{sw} = specific gravity of seawater (1.03), G_s = specific gravity of solids, C_{sw} = specific heat of seawater (0.94 cal/gr-^{o}C), C_s = specific heat of solids (0.18 $\frac{cal}{gr\ ^oC}$) and V_T = total volume. The units for Equation 15 are cal/cm^3-^{o}C. Equation 15 has been plotted in Fig. 11. Substituting Equation 15 into Equation 11 for the thermal diffusivity leaves only the thermal conductivity unknwon, since the porosity is known for each load increment.

If the temperature rise of the transducer versus time is known, the thermal conductivity of the soil can be backfigured using Equation 11. This is obtained by recording the change of resistance of the small bead thermister glued to the probe. A strip chart recorder was used to continuously record this change during the test. Due to the complexity of the equation defining the temperature rise of the transducer, a FORTRAN computer program was written that iterates to the value of thermal conductivity. The program used is listed in Appendix II.
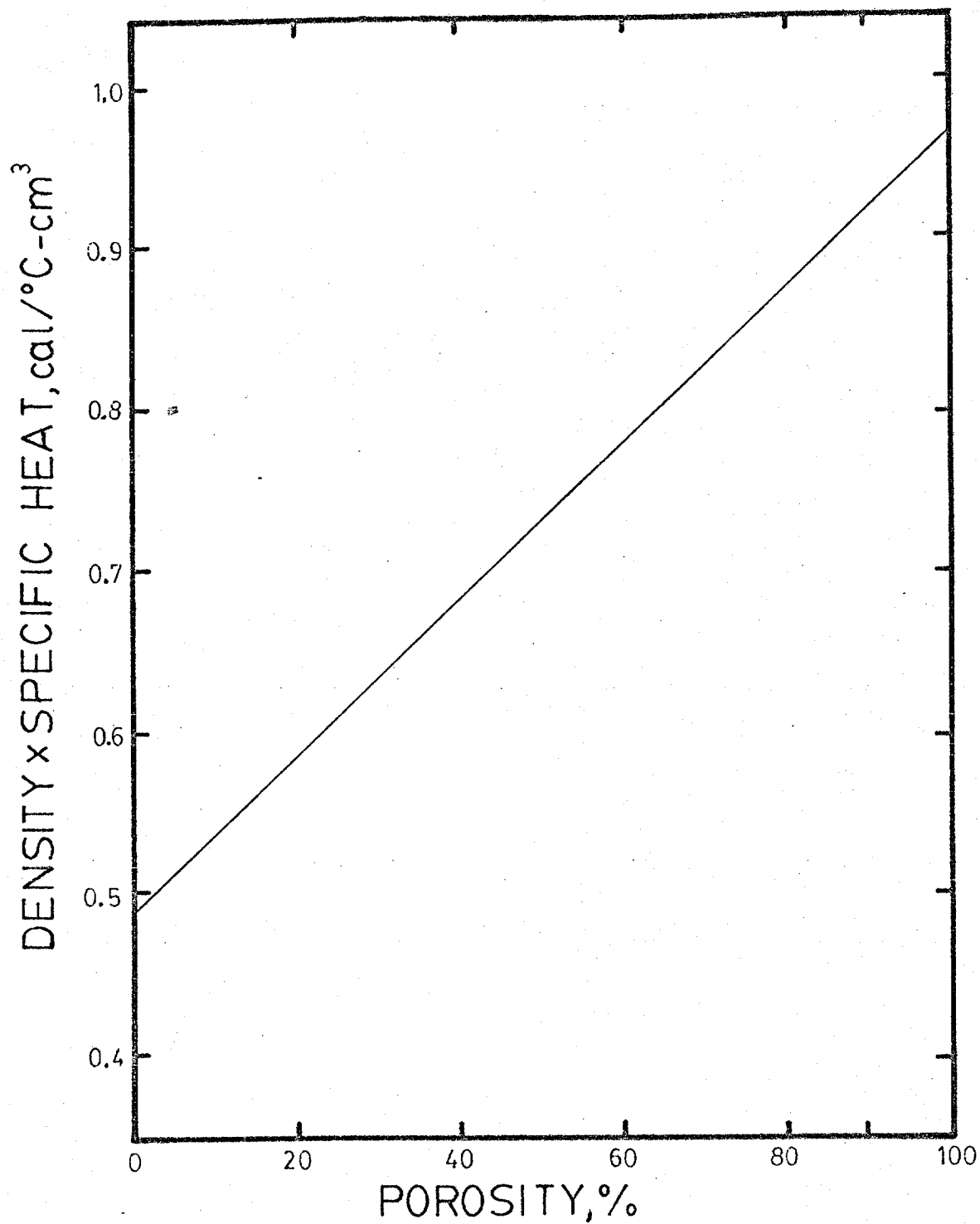
FIG. 11.-Density times specific heat for a soil-water mixture

TEST EQUIPMENT AND PROCEDURE

Test Equipment

To measure the thermal conductivity at various porosities required a load application system that was able to achieve a load of 10,000 psi (68.9 MPa) and a containment vessel capable of withstanding these pressures. The containment vessel also had to withstand corrosion due to saltwater that was mixed with the soils.

A 2.5 inch (13.9 cm.) consolidometer chamber was constructed out of 316 stainless steel one inch (2.54 cm.) thick. The sediment was loaded by a piston as shown in Fig. 12. The piston is sealed by a U-shaped teflon seal manufactured by Flourocarbon Company of California, having virtually no frictional resistance. Located at the top and bottom of the soil sample are porous stones to facilitate drainage of the pore water. The water that was expelled when a load was applied was collected into two burets and measured to determine the change in volume of the sediment-saltwater system. The change in volume of the sediment-saltwater system was also checked by measuring the motion of the piston using a dial gage. Once the volume change was known, the porosity was calculated at the end of each load incremnet.

Located at the centerline of the sediment-saltwater system inside the consolidation chamber was the thermal conductivity transducer. The transducer was attached to four wires and then sealed with a thin coat of protective covering to keep put saltwater. The wires were then led out the bottom drain hole of the consolidometer to the signal analyzer. To prevent water leakage where the wires left the consolidometer, a
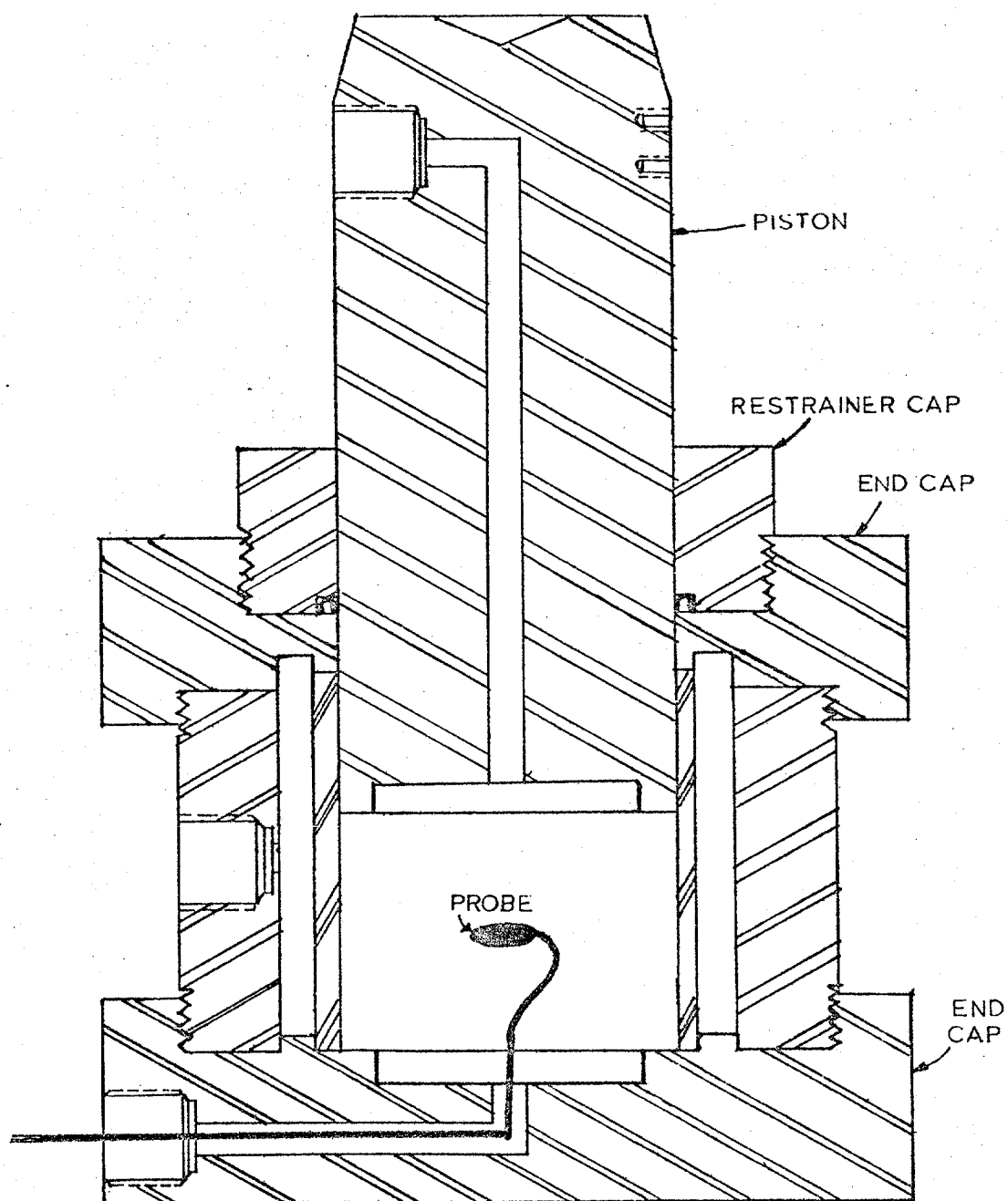
FIG.12.-Consolidometer with the thermal conductivity transducer

rubber grommet was used.

A deadweight lever system was used to develop the high pressures necessary for consolidation. The compound lever system magnified the applied force eighty times. The deadweight lever system had many advantages:

1) No calibration was needed to find the applied force. Only the distance to each fulcrum was needed.

2) No hydraulic hoses, regulators or valves were needed since the load application was by deadweights.

3) The major advantage was the low cost of the system. There was no expensive equipment necessary, as would have been required, had a hydraulic system been used.

A schematic diagram of the system is shown in Fig. 13. Several pictures of the consolidometer, lever system and the thermal conductivity test equipment are shown is Figs. 14 through 16.
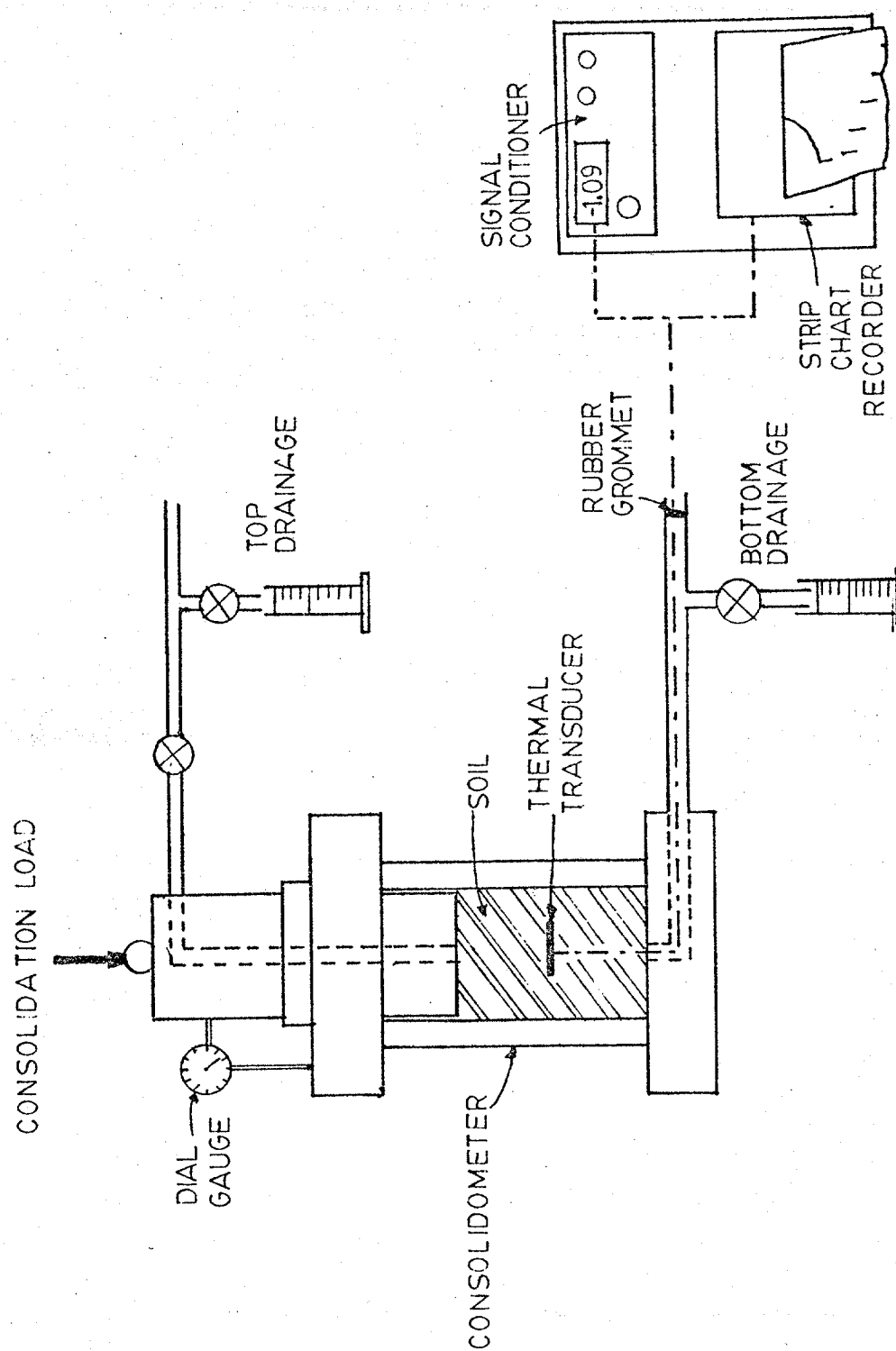
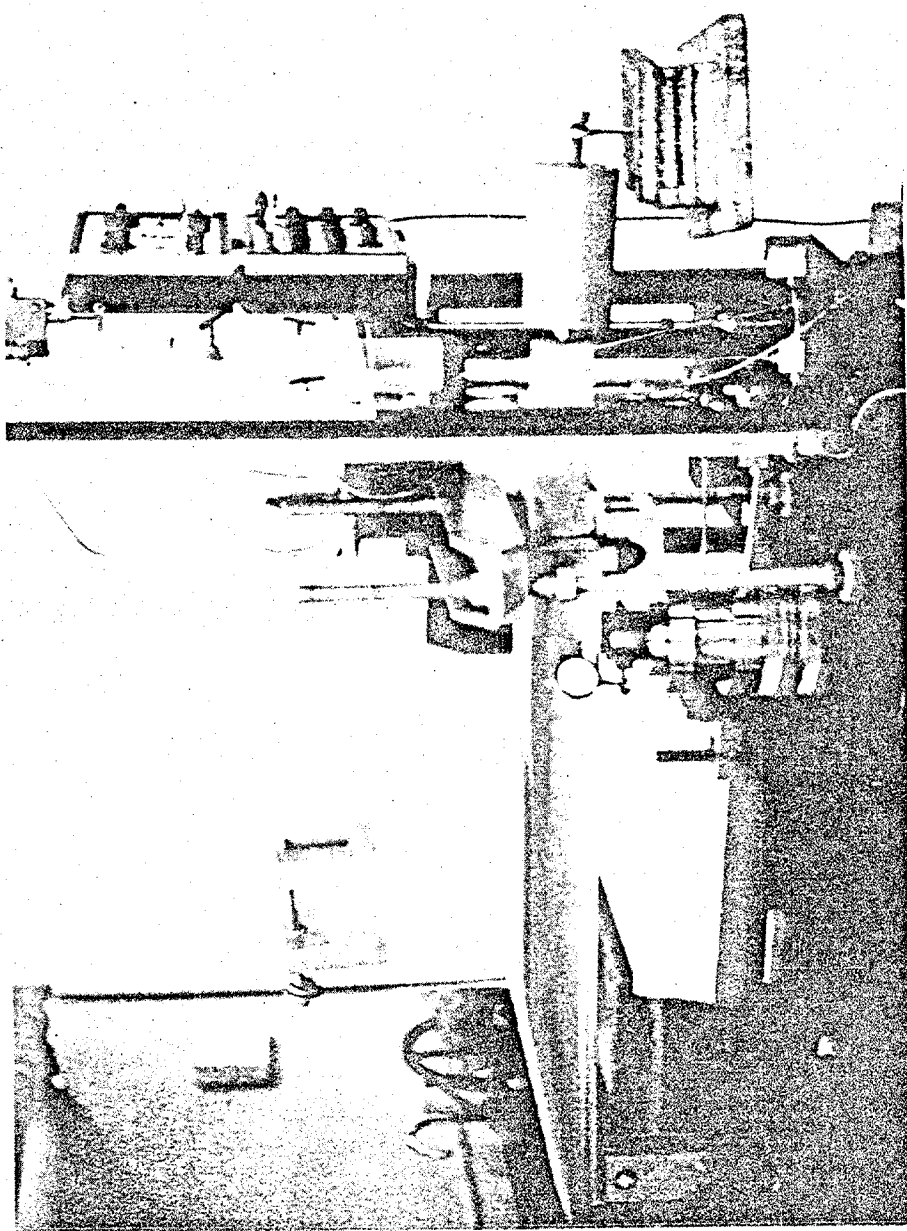FIG.13.-Schematic of consolidometer and heat conductivity equipment
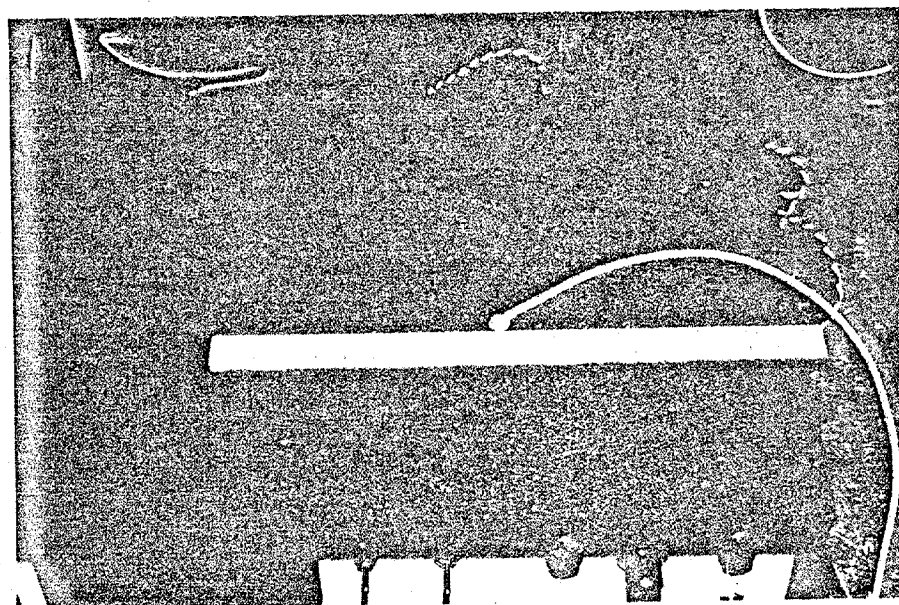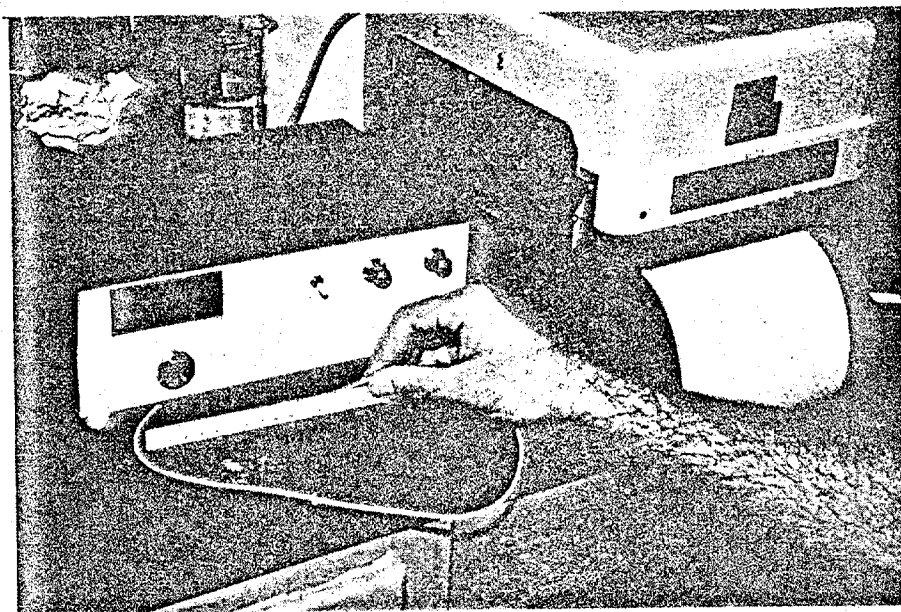
FIG.14.-Side view of the lever system

FIG.16.-Heat conductivity transducer and equipment

Test Procedure

The consolidation tests performed for this study simulated the progressive burial of a sediment from a high initial porosity approximating conditions that would be expected at the midline to a low porosity that resulted from the application of 10,000 psi (69.9MPa). The soil sample was mixed with reconstituted seawater to form a sample of around 75% porosity. This corresponded to a moisture content of about 125%. The soil sample was then allowed to sit overnight to assure that the soil particles were at a uniform moisture content.

The consolidation test was started with the soil sample around 2 inches (5.1 cm.) high. The thermal conductivity transducer was carefully positioned in the middle of the sample. This was done to insure equal heat flow from both top and bottom of the transducer. The load was applied by hanging weights on the front hanger of the lever lever system. The change in sample height was measured by a dial gage and was checked by collecting the water expelled from the soil into burets. When the change in sample height leveled off, the consolidation process was assumed complete. The porosity at the end of each consolidation increment was calculated from the following formulas:

$$\Delta e = e_f - e_i = \frac{\Delta h(1+e_i)}{h} \quad \ldots \ldots \ldots \ldots \ldots \ldots (16)$$

and

$$n = \frac{e_f}{1+e_f} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (17)$$

where Δe = change of void ratio; Δh = change in sample height; e_i = original void ratio; h = sample height before the load was applied; e_f =

void ratio at the end of each consolidation increment; and n = porosity at the end of the load increment.

At the end of each consolidation test the thermal conductivity was determined. The consolidation load was kept constant while energy was supplied to the heater. The temperature rise of the probe was measured for six minutes and was recorded by a strip chart recorder. Using the time-temperature data and Equation 11 the thermal conductivity was calculated for each load increment.

PRESENTATION AND DISCUSSION OF RESULTS

Soil Sample Description

The soil samples used for this study were Kaolinite clay and 20-30 mesh Ottawa sand. They were originally in dry form but seawater was added to obtain initial conditions approximating those expected at the mudline. The Kaolinite was mixed with seawater to obtain a porosity of 70 percent. However, the Ottawa sand was poured into a jar of salt-water and allowed to settle, obtaining the highest porosity expected in the ocean. The Ottawa sand was then densified by vibrating the jar. Only two different values for the thermal conductivity of Ottawa sand were obtained: one in an uncompacted state and the other in a dense state.

Test Curves

Under each consolidation load the sample height changed as the saltwater was expelled. The change in sample height under each load was plotted against time. A typical curve illustrating the change in sample height with respect to time is shown in Fig. 17. The consolidation load for Kaolinite varied from 63 psi (433 kPa) to 10,386 psi (71.6 MPa). The final porosity for each consolidation load is tabulated in Table 1 and plotted against the consolidation load on a log-log plot as shown in Fig. 18.

The plot of consolidation load versus porosity can be described by a "power law" of the form

$$\sigma = An^B \quad\quad\quad\quad\quad\quad\quad (18)$$

where σ = consolidation pressure when the pore pressure is zero and n = the final porosity. When a "power law" function is plotted on log-log paper the function becomes a straight line. On the log-log plot of Equation 18, the coefficient A is the intercept when n = 1% and the coefficient B is the slope of the straight line. The compressibility constants A and B were found to be different for different soils. The power law fit the data quite well, giving a correlation coefficient of 0.90.

At the end of each consolidation load a given amount of power was supplied to the heater in the transducer and the change in resistance of the thermister was recorded with respect to time. The amount of power that was supplied to the transducer was 0.1 $\frac{cal}{sec}$. The applied power was produced by 7.93 volts and 52.5 milliamps. For each consolidation load
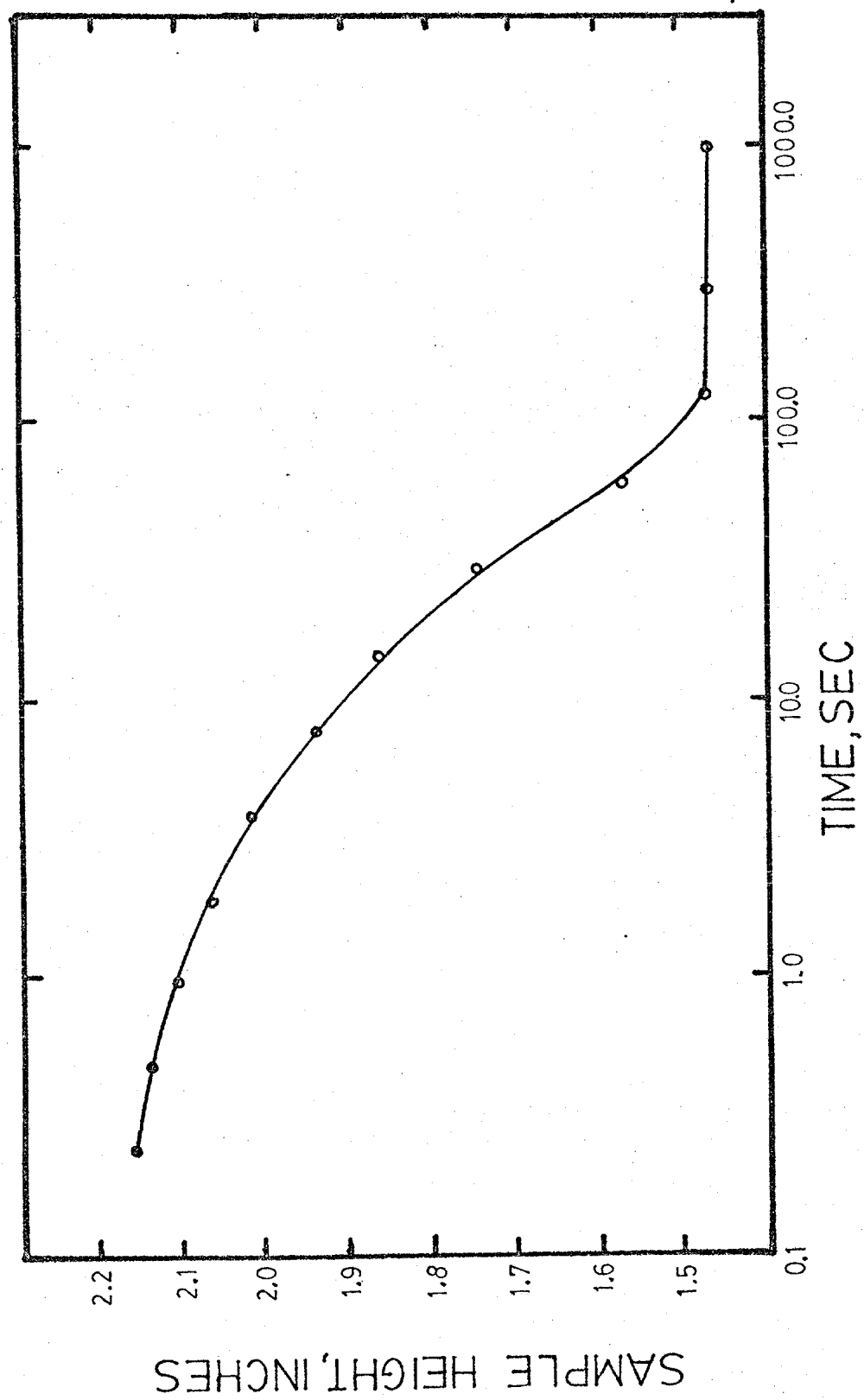
TIME,SEC

FIG.17.-Sample height versus time for Kaolinite clay
under 63 psi consolidation load

TABLE 1.-Consolidation Load, Porosity and Thermal
Conductivity for Kaolinite Clay and Ottawa Sand.

| Material	Consolidation Load(psi)	Porosity	Thermal Conductivity (cal/^{o}C-cm-sec)
Kaolinite Clay	0	69.5	0.79×10^{-3}
	63	53.2	0.96×10^{-3}
	99	50.9	1.05×10^{-3}
	206	46.9	1.10×10^{-3}
	350	43.8	1.15×10^{-3}
	637	40.0	1.17×10^{-3}
	1210	36.0	1.18×10^{-3}
	2357	31.1	1.19×10^{-3}
	350 unload	32.5	1.19×10^{-3}
	63 unload	37.1	1.19×10^{-3}
	206 reload	36.8	1.19×10^{-3}
	637 reload	34.7	1.19×10^{-3}
	2357 reload	29.8	1.19×10^{-3}
	4651	25.1	1.21×10^{-3}
	10386	14.7	1.21×10^{-3}
Ottawa Sand	Uncompacted	39.0	1.39×10^{-3}
	Compacted	36.0	1.49×10^{-3}
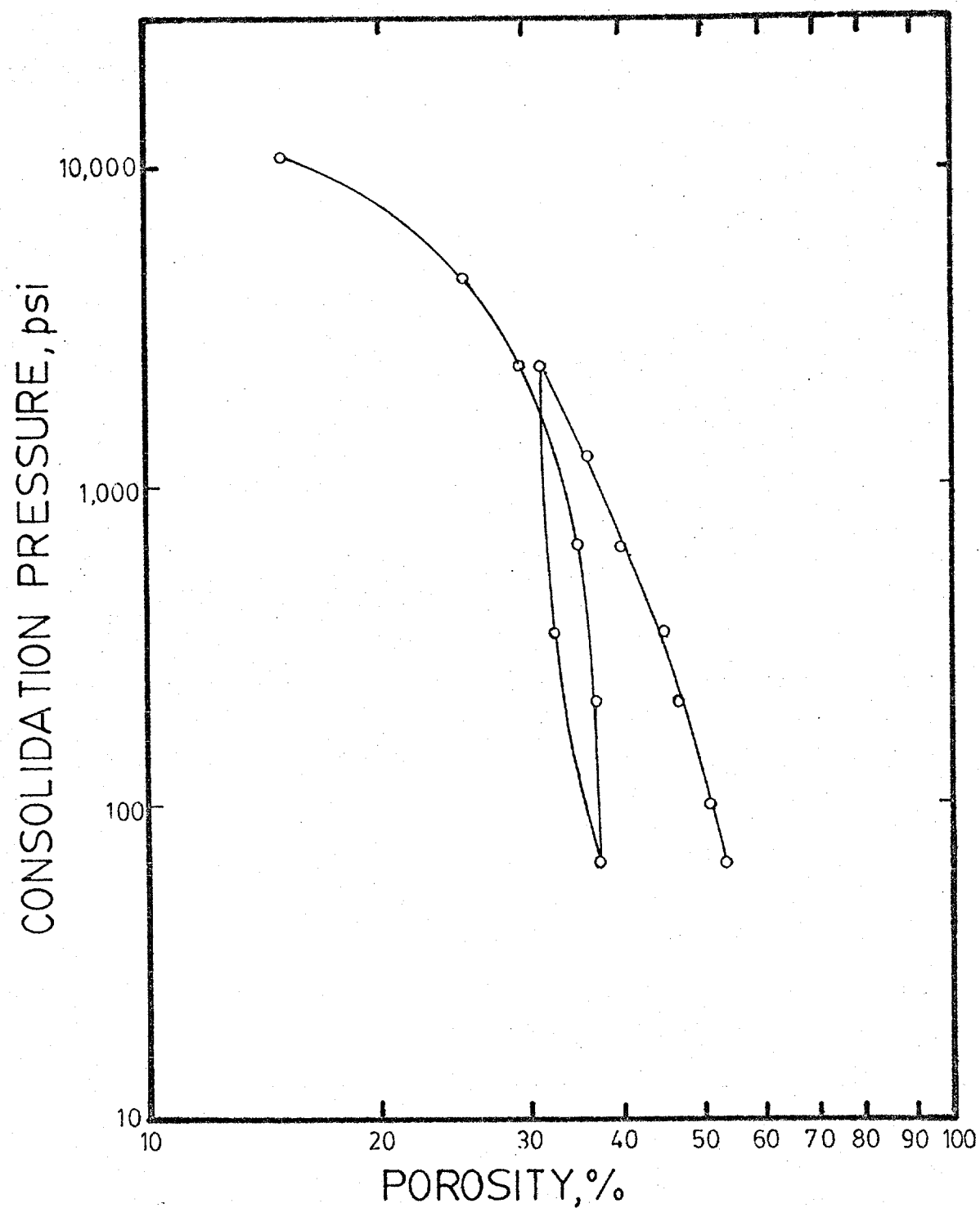
FIG.18.-Consolidation load versus porosity for Kaolinite clay

the change in resistance of the thermister in the transducer was recorded on a strip chart recorder. The resistance of the thermister was converted to temperature using the thermister calibration curve shown in Fig. 19. A typical curve of temperature rise of the transducer with respect to time is shown in Fig. 20.

The thermal conductivity of the soil was determined by using the computer program listed in Appendix II. Various temperatures and the corresponding times were inputted to the computer program. The specific heat and density were calculated using Equation 15 with the respective porosity. For each data point the computer program, using Equation 11, solved for the thermal conductivity. It was found that the thermal conductivity calculated by the computer program varied for early times and converged to certain value after three minutes. A plot of the thermal conductivity with respect to time is shown in Fig. 21. The thermal conductivity for this particular consolidation load was 0.79×10^{-3} $\frac{cal}{cm-sec}$. Using this value of thermal conductivity in Equation 11, the temperature rise was calculated for different time intervals. This is the dashed curve shown in Fig. 20. It can be seen that the transducer does not predict the early temperatures very well. It is believed that this is due to the fact the transducer is not a spherical source.

Table 1 lists the values for the thermal conductivity for Kaolinite clay and Ottawa sand at their respective porosities. A curve of thermal conductivity versus porosity for Kaolinite clay is shown in Fig. 22. The best fit curve determined by a least squares approximation relating porosity and thermal conductivity for Kaolinite clay can be described by a parabolic equation. The equation was
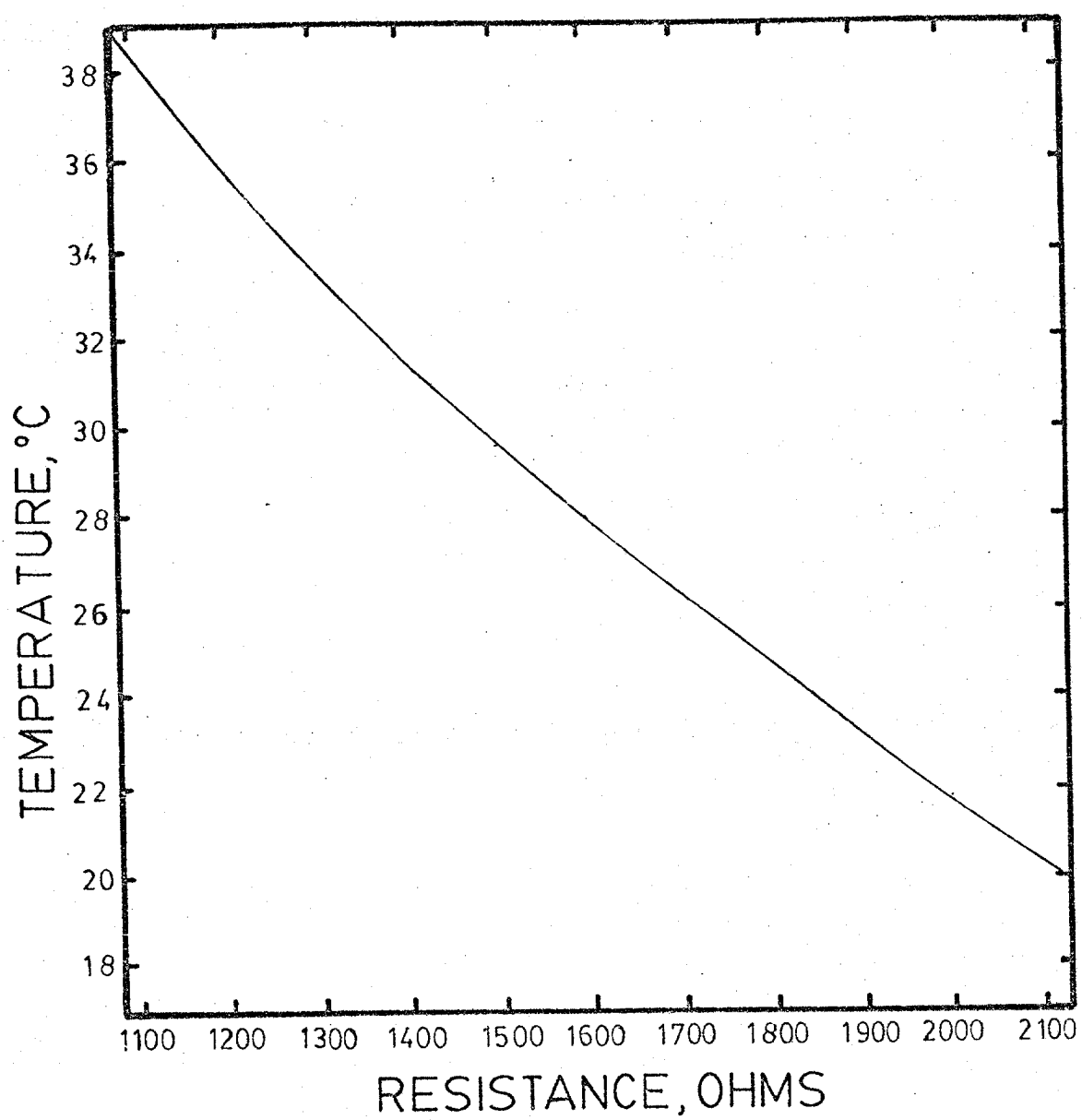
FIG.19.-Thermister calibration curve
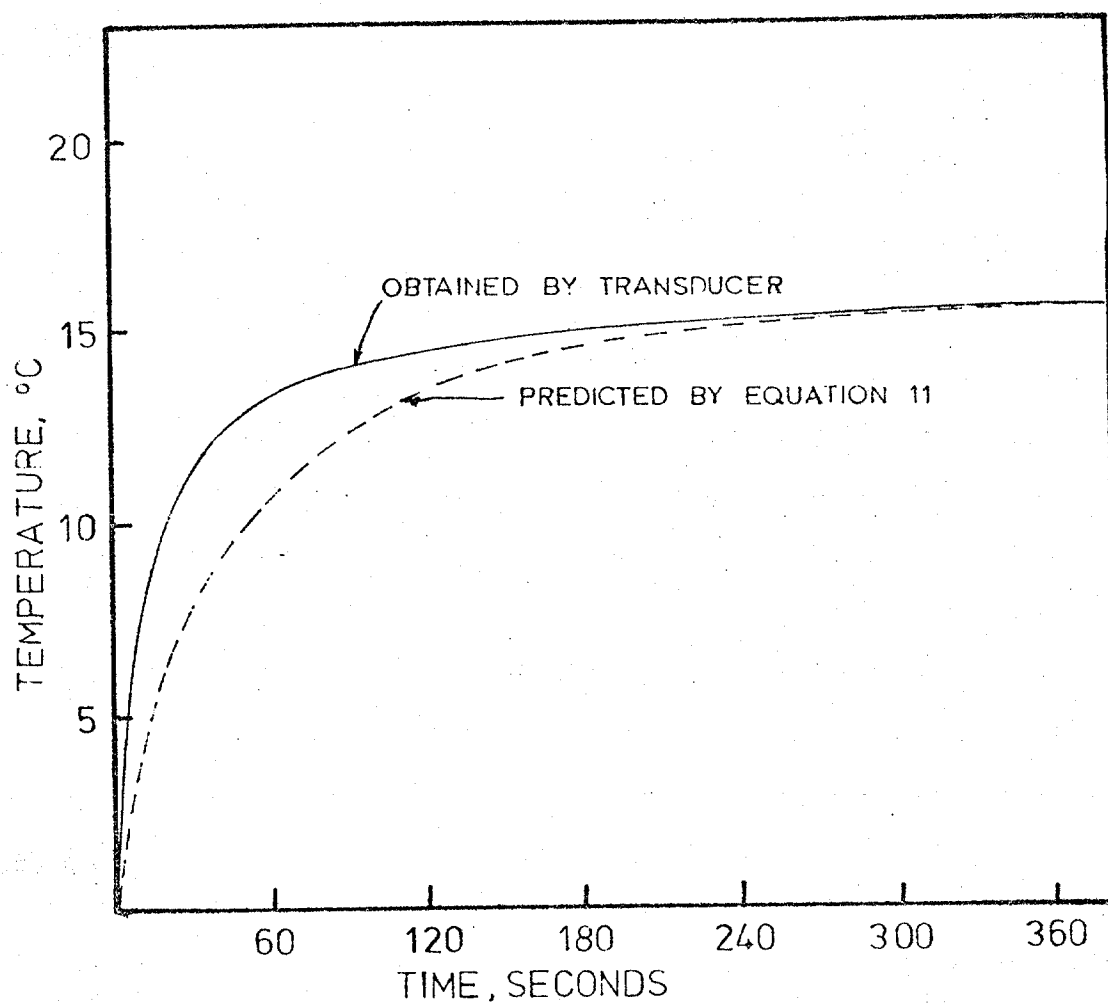
FIG. 20.- Comparison between the temperature of the transducer and the temperature predicted by Equation 11

FIG.21.-Change in thermal conductivity
with respect to time

$$k = 1.14 \times 10^{-3} + 7.78 \times 10^{-6} n - 1.87 \times 10^{-7} n^2 \quad . \quad .(19)$$

where k = thermal conductivity in cal/^{0}C-cm-sec and n = porosity in percent. This equation does not use the unloading and reloading data points.

FIG.22.-Thermal conductivity versus porosity for Kaolinite clay

Discussion of Results

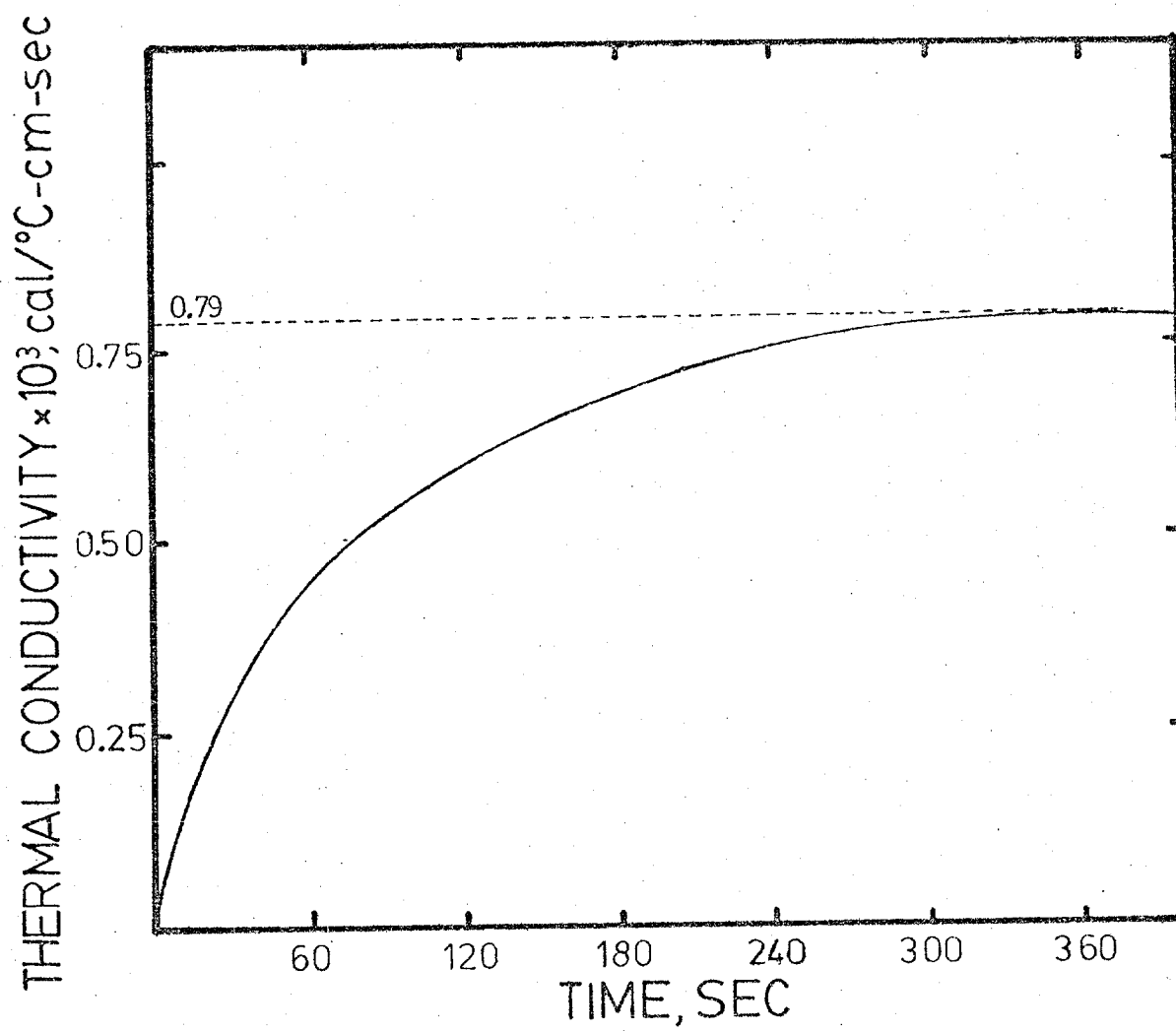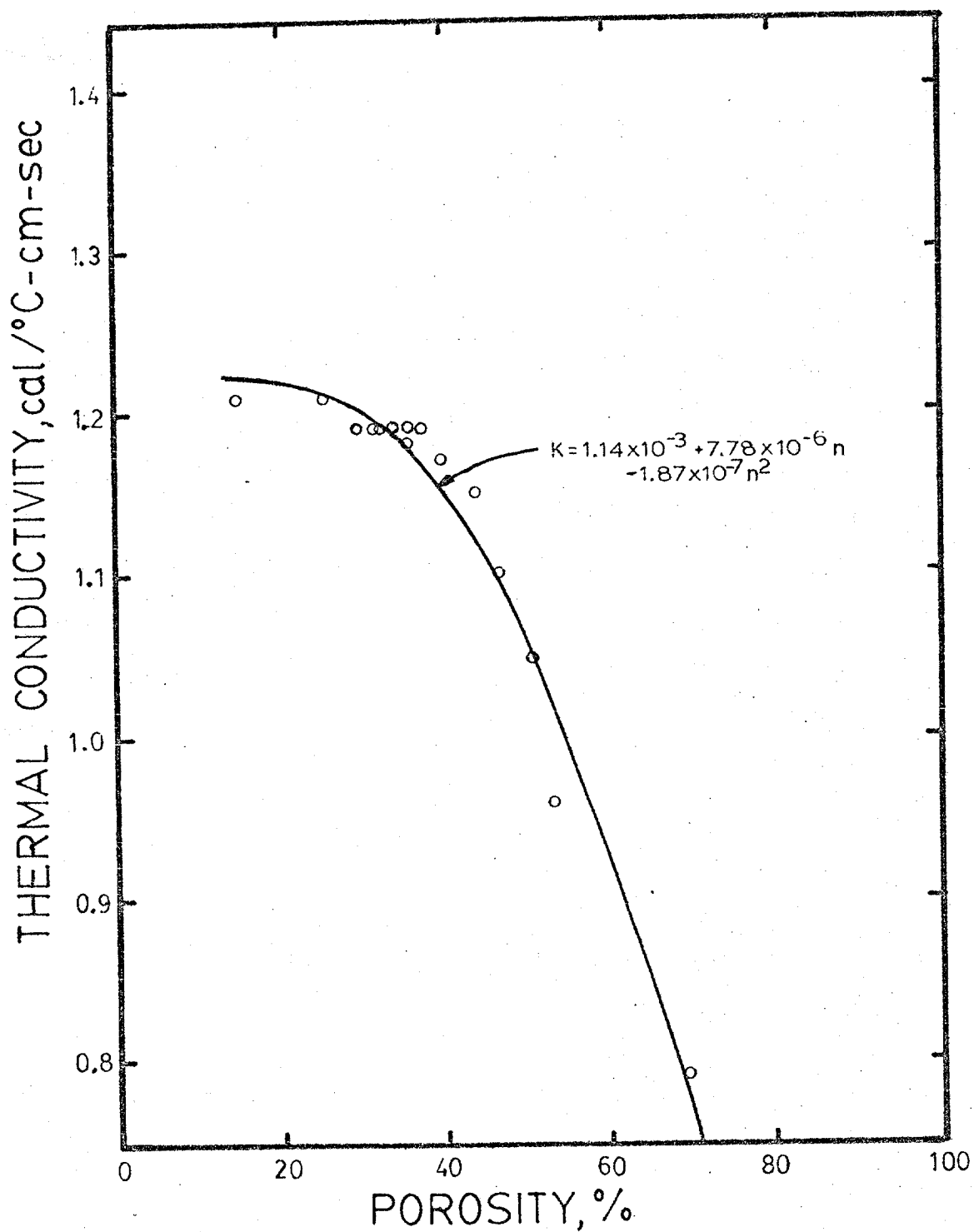Values for the thermal conductivity of the soils tested ranged from 0.79 x 10^{-3} cal/^{o}C-cm-sec at 69% porosity to 1.21 x 10^{-3} cal/^{o}C-cm-sec at 15% porosity. The greatest change occurred at the low loads when the porosity changed rapidly with small increases in load.

If the values for the thermal conductivity of Kaolinite clay used in this experiment are compared with values for marine clay from the literature, it is found that the literature reports values 1.5 to 2.0 times as large. It was not known why the values differed by this amount.

The literature was re-examined to determine if any values for the thermal conductivity of Kaolinite clay existed. It was found that Kaolinite was used as a binder in road subgrades and the thermal conductivity for this mixture had been determined. However, Kaolinite was only added in limited amounts of 10-15% (11). It was also found that bricks manufactured from Kaolinite had values for the thermal conductivity considerably less than bricks made from other clays (red clay in particular).

It was finally found that clay-water mixtures exhibited anisotropic thermal conductivity similar to anisotropic water permeability. Penner determined the thermal conductivity for clay soils and found that it was indeed anisotropic (22). He took samples at various depths in the earth and found that the vertical thermal conductivity ranged from 1.1 to 1.7 less than the horizontal thermal conductivity. He explained this by the nature of clay particle orientation. At high porosities the soil-water matrix is in a flocculated state. As the soil is compressed the particles tend to align themselves. This can be seen

in Fig. 23. Heat flows much easier horizontally than vertically. This is also true for the permeability of water for soils.
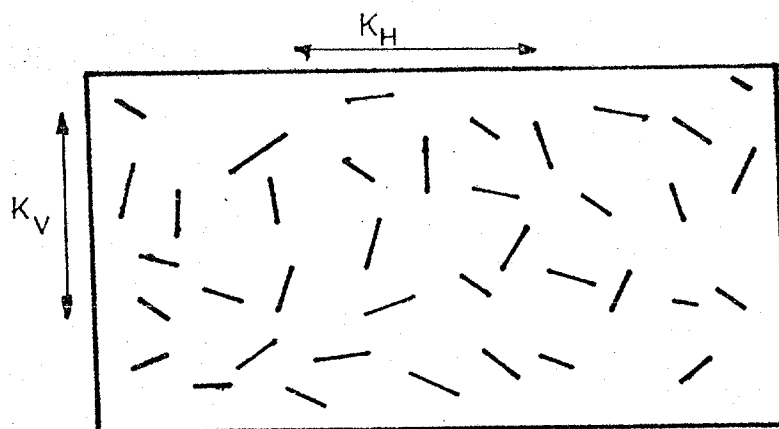
CASE I-SOIL IN A FLOCCULATED STATE
$$K_{VERTICAL} = K_{HORIZONTAL}$$
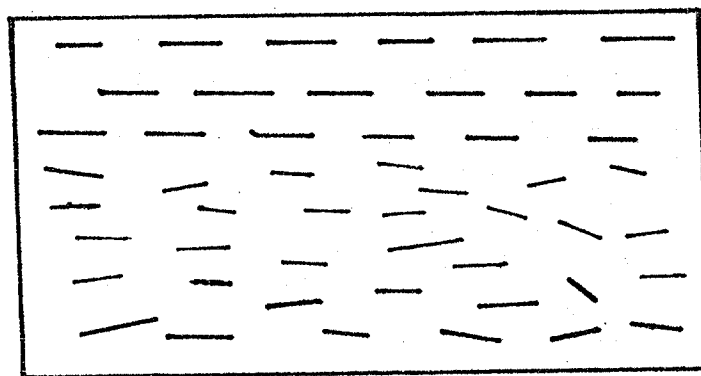
CASE II-SOIL AFTER CONSOLIDATION LOAD
$$K_{HORIZONTAL} >> K_{VERTICAL}$$

FIG.23.-Variation in thermal conductivity with particle alignment

CONCLUSIONS AND RECOMMENDATIONS

A new apparatus was developed to determine the thermal conductivity of saturated soils under various consolidation pressures. Due to the geometry of the transducer, the vertical thermal conductivity was measured in these tests. Based on the results obtained, the following was concluded:

1. The spherical heat equation derived by Jaeger can be used for a flat disc without loss of accuracy, if an effective radius for the disc is found.

2. Using the transducer, the thermal conductivity of Ottawa sand, both saturated and dry, was determined, and the results agreed quite well with values reported in the literature.

3. The values obtained for the thermal conductivity of Kaolinite clay were considerably lower than for other saturated clays found in the literature. The large applied consolidation loads cause clay particle alignment to take place. It is believed the low vertical thermal conductivity is due to anisotropic orientation of the clay particles.

4. A parabolic equation is a good model for the relationship between porosity and thermal conductivity for Kaolinite clay.

5. For Kaolinite, the thermal conductivity does not change when the sample is unloaded and allowed to rebound. Therefore, it seems that the thermal conductivity depends more upon particle alignment than upon porosity when the Kaolinite is at low porosities.

Further work should be done to include more soils. Different mineralogical content might help establish a relationship between the

Atterberg limits and the thermal conductivity for different soils. Also, since clay soils have anistropic particle orientation, an analysis that accounted for this affect could be done. This would involve using two transducers positioned 90° apart. Since Equation 11 was used to calculate the thermal conductivity for an anistropic soil, a better prediction of temperature as a function of time might be obtained by considering the horizontal thermal conductivity as a multiple of the vertical thermal conductivity. This should improve the correlation between calcluated and experimentally determined temperatures as shown in Fig. 20. The solution for the temperature at a point source in an infinite, anisotropic body is given by Carslaw and Jeager on page 257(7).

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APPENDIX II. - COMPUTER PROGRAM

INSTRUCTIONS FOR USE OF THE COMPUTER PROGRAM TO

SOLVE FOR THE THERMAL CONDUCTIVITY

The following computer program was written in FORTRAN language and will run on any computer capable of compiling this language. All input to the program is "free form" input. This means that the data may go anywhere on the computer card as long as it is in the correct order and there is a space between each inputted variable. If more than one set of data is to be calculated all that is needed is to stack the following data sets back to back.

Instructions

    I. First Data Card
          This card is a soil description card. Anything may be put on this card as long as it is only 80 spaces long. Only one card can be used, however.

    II. Second Data Card
          The second card contains all information regarding the thermal transducer. Three variables must be on this card in the following order; transducer voltage, transducer amperage, transducer radius. The following units and dimensions must be used.
              1. voltage-volts
              2. amperage-milliamps
              3. radius-centimeters

    III. Third Data Card
          This card is a soil variable card. On this card the following must be inputted; the porosity at the load for which the thermal conductivity is to be calculated (in percent), the number of time-temperature points for this load increment and the specific gravity of the soil.

    IV. Remaining Data Crads
          The remaining data cards contain the respective times and temperatures obtained by the thermal transducer. There should be only one set of time and temperature on each data card. The time should be in seconds and the temperature in degrees celsius. There should be as many of these cards as specified on card three.

The following example will illustrate the correct input to the program.

Card I   - Soil is Kaolinite Clay-Load was 63 psi
Card II   - 7.93 52.5 0.41
Card III   - 53.20 5 2.64
Card IV   - 120 12.5
Card V   - 180 12.7
Card VI   - 240 12.9   5 cards as specified on Card III
Card VII   - 300 13.0
Card VIII- 360 13.0

```
      DIMENSION TITE(15,2),ILINE(20)
      REAL KO,KF
100   READ(5,41,END=300)  (ILINE(I),I=1,20)
 41   FORMAT (20A4)
      WRITE(6,40)  (ILINE(I),I=1,20)
 40   FORMAT (///,20A4)
      READ(5,*) V,A,R
      WRITE(6,50) V,A,R
 50   FORMAT(10X,'VOLTAGE =',F5.2,1X,'VOLTS',10X,'AMPERAGE =',F7.2,
     *1X,'MA',10X,'PROBE RADIUS IS ',F8.3,' CM')
      READ(5,*) POR,N,SPG
C CALCULATE THE MOISTURE CONTENT W
      W=POR*64./62.4/((100.-POR)*SPG)
      WRITE(6,51) W,POR
 51   FORMAT(10X,'MOISTURE CONTENT =',F6.4,5X,'POROSITY = ',F6.2)
      DO 1 I=1,N
      READ(5,*) TITE(I,1),TITE(I,2)
  1   CONTINUE
      EPSI=.000001
C CALCULATE THE SPECIFIC HEAT MULTIPLIED BY THE DENSITY
      CRHO10=POR*1.03*0.94/100.+(100.-POR)*SPG*0.18/100.
C CALCULATE THE POWER GOING TO THE PROBE
      Q=V*A*.2389/1000.
C CALCULATE THE CONSTANT Q DIVIDED BY 4*PHI*RADIUS
      PHI=4.*ATAN(1.)
      C1=Q/(PHI*4.*R)
      DO 2 I=1,N
C CALCULATE INITIAL THERMAL CONDUCTIVITY, HAVE TO START SOMEWHERE
C INITIAL STARTING POINT WILL BE C1 DIVIDED BY TWICE THE TEMPERATURE
      KO=C1/(TITE(I,2)*2.)
C KO=INITIAL THERMAL CONDUCTIVITY
  3   ALPHA=KO*CRHO10
      C3=ALPHA*TITE(I,1)/(R*R)
      C2=SQRT(C3)
      CALL CERFC(C2,ERFC)
      KF=C1/TITE(I,2)*(1.-EXP(C3))*ERFC)
      DEPSI=ABS(KF-KO)
      IF (DEPSI.LE.EPSI) GO TO 10
      KO=KF
      GO TO 3
 10   WRITE(6,52) KF,TITE(I,1),TITE(I,2)
 52   FORMAT(10X,'THERMAL CONDUCTIVITY =',E11.3,5X,'TIME =',F7.2,
     *5X,'TEMPERATURE =',F7.2)
  2   CONTINUE
      GO TO 100
300   STOP
      END
```

```fortran
C THE FOLLOWING SUBROUTINE CALCULATES THE COMPLIMENTARY ERROR FUNCTION
      SUBROUTINE CERFC(C2,ERFC)
      REAL M
      F5N=0.0
      DN=1.0
      DO 1 I=1,10
      N=2*(I-1)+1
      NI=I-1
      M=N
      F2N=C2**N
      F3N=F2N*2**NI
      DN=M*DN
      F4N=F3N/DN
      F5N=F5N+F4N
    1 CONTINUE
      PHI=4.*ATAN(1.0)
      SPHI=SQRT(PHI)
      ERFC=1.-(2./SPHI*F5N*EXP(-(C2**2)))
      RETURN
      END
```

VITA

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The typist for this thesis was Mrs. Joy Taylor.